Game Theory

Lecture Notes By

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Bayesian Mechanisms

Note: *This is a only a draft version, so there could be flaws. If you find any errors, please do send email to* `hari@csa.iisc.ernet.in`. *A more thorough version would be available soon in this space.*

1 Bayesian Implementation: The dAGVA Mechanism

Recall that we mentioned two possible routes to get around the Gibbard–Satterthwaite Impossibility Theorem. The first was to focus on restricted environments like the quasilinear environment, and the second one was to weaken the implementation concept and look for an SCF which is ex-post efficient, nondictatorial, and Bayesian incentive compatible. In this section, our objective is to explore the second route.

Throughout this section, we will once again be working with the quasilinear environment. As we saw earlier, the quasilinear environments have a nice property that every social choice function in these environments is nondictatorial. Therefore, while working within a quasilinear environment, we do not have to worry about the nondictatorial part of the social choice function. We can just investigate whether there exists any SCF in quasilinear environment, which is both *ex-post efficient* and *BIC*, or equivalently, which has three properties — *AE, BB,* and *BIC*. Recall that in the previous section, we have already addressed the question whether there exists any SCF in quasilinear environments that is *AE, BB*, and *DSIC*, and we found that no function satisfies all these three properties. On the contrary, in this section, we will show that a wide range of SCFs in quasilinear environments satisfy three properties — AE, BB, and BIC.

1.1 The dAGVA Mechanism

The following theorem, due to d'Aspremont and Gérard-Varet [1] and Arrow [2] confirms that in quasilinear environments, there exist social choice functions that are both ex-post efficient and Bayesian incentive compatible. We refer to this theorem as the *dAGVA* Theorem.

Theorem 1.1 (The dAGVA Theorem) *Let the social choice function $f(\cdot) = (k^*(\cdot), t_1(\cdot), \ldots, t_n(\cdot))$ be allocatively efficient and the agents' types be statistically independent of each other (i.e. the density $\phi(\cdot)$ has the form $\phi_1(\cdot) \times \ldots \times \phi_n(\cdot)$). This function can be truthfully implemented in Bayesian Nash* *equilibrium if it satisfies the following payment structure, known as the dAGVA payment (incentive) scheme:*

$$t_i(\theta) = E_{\tilde{\theta}_{-i}}\left[\sum_{j\neq i} v_j(k^*(\theta_i, \tilde{\theta}_{-i}), \tilde{\theta}_j)\right] + h_i(\theta_{-i}) \;\; \forall\, i = 1, \ldots, n; \quad \forall \theta \in \Theta \tag{1}$$

where $h_i(\cdot)$ is any arbitrary function of θ_{-i}.

Proof: Let the social choice function $f(\cdot) = (k^*(\cdot), t_1(\cdot), \ldots, t_n(\cdot))$ be allocatively efficient, i.e., it satisfies the condition (**??**) and also satisfies the dAGVA payment scheme (1). Consider

$$E_{\theta_{-i}}\left[u_i(f(\theta_i, \theta_{-i}), \theta_i)|\theta_i\right] \;\; = \;\; E_{\theta_{-i}}\left[v_i(k^*(\theta_i, \theta_{-i}), \theta_i) + t_i(\theta_i, \theta_{-i})|\theta_i\right].$$

Since θ_i and θ_{-i} are statistically independent, the expectation can be taken without conditioning on θ_i. This will give us

$$\begin{aligned} E_{\theta_{-i}}\left[u_i(f(\theta_i, \theta_{-i}), \theta_i)|\theta_i\right] &= E_{\theta_{-i}}\left[v_i(k^*(\theta_i, \theta_{-i}), \theta_i) + h_i(\theta_{-i}) + E_{\tilde{\theta}_{-i}}\left[\sum_{j\neq i} v_j(k^*(\theta_i, \tilde{\theta}_{-i}), \tilde{\theta}_j)\right]\right] \\ &= E_{\theta_{-i}}\left[\sum_{j=1}^{n} v_j(k^*(\theta_i, \theta_{-i}), \theta_j)\right] + E_{\theta_{-i}}[h_i(\theta_{-i})]. \end{aligned}$$

Since $k^*(\cdot)$ satisfies the condition (**??**),

$$\sum_{j=1}^{n} v_j(k^*(\theta_i, \theta_{-i}), \theta_j) \geq \sum_{j=1}^{n} v_j(k^*(\hat{\theta}_i, \theta_{-i}), \theta_j) \;\forall\; \hat{\theta}_i \in \Theta_i.$$

Thus we get, $\forall\;\; \hat{\theta}_i \in \Theta_i$

$$E_{\theta_{-i}}\left[\sum_{j=1}^{n} v_j(k^*(\theta_i, \theta_{-i}), \theta_j)\right] + E_{\theta_{-i}}[h_i(\theta_{-i})] \geq E_{\theta_{-i}}\left[\sum_{j=1}^{n} v_j(k^*(\hat{\theta}_i, \theta_{-i}), \theta_j)\right] + E_{\theta_{-i}}[h_i(\theta_{-i})].$$

Again by making use of statistical independence we can rewrite the above inequality in the following form

$$E_{\theta_{-i}}\left[u_i(f(\theta_i, \theta_{-i}), \theta_i)|\theta_i\right] \geq E_{\theta_{-i}}\left[u_i(f(\hat{\theta}_i, \theta_{-i}), \theta_i)|\theta_i\right] \;\; \forall\; \hat{\theta}_i \in \Theta_i.$$

This shows that when agents $j \neq i$ announce their types truthfully, agent i finds that truth revelation is his optimal strategy, thus proving that the SCF is BIC.

Q.E.D.

After the results of d'Aspremont and Gérard-Varet [1] and Arrow [2], a direct revelation mechanism in which the SCF is allocatively efficient and satisfies the dAGVA payment scheme is called as *dAGVA mechanism/expected externality mechanism/expected Groves mechanism.*

Definition 1.1 (dAGVA/expected externality/expected Groves Mechanisms) *A direct revelation mechanism, $\mathscr{D} = ((\Theta_i)_{i\in N}, f(\cdot))$ in which $f(\cdot) = (k(\cdot), t_1(\cdot), \ldots, t_n(\cdot))$ satisfies (**??**) and (1) is known as dAGVA/expected externality/expected Groves Mechanism.*[1]

[1] *We will sometimes abuse the terminology and simply refer to a SCF $f(\cdot)$ satisfying (**??**) and (1) as dAGVA/expected externality/expected Groves Mechanism.*

1.2 The dAGVA Mechanism and Budget Balance

We now show that the functions $h_i(\cdot)$ above can be chosen to guarantee $\sum_{i=1}^{n} t_i(\theta) = 0$. Let us define,

$$
\begin{aligned}
\xi_i(\theta_i) &= E_{\tilde{\theta}_{-i}} \left[\sum_{j \neq i} v_j(k^*(\theta_i, \tilde{\theta}_{-i}), \tilde{\theta}_j) \right] \ \forall\, i = 1, \ldots, n \\
h_i(\theta_{-i}) &= -\left(\frac{1}{n-1} \right) \sum_{j \neq i} \xi_j(\theta_j) \ \ \forall\, i = 1, \ldots, n.
\end{aligned}
$$

In view of the above definitions, we can say that

$$
\begin{aligned}
t_i(\theta) &= \xi_i(\theta_i) - \left(\frac{1}{n-1} \right) \sum_{j \neq i} \xi_j(\theta_j) \\
\Rightarrow \sum_{i=1}^{n} t_i(\theta) &= \sum_{i=1}^{n} \xi_i(\theta_i) - \left(\frac{1}{n-1} \right) \sum_{i=1}^{n} \sum_{j \neq i} \xi_j(\theta_j) \\
\Rightarrow \sum_{i=1}^{n} t_i(\theta) &= \sum_{i=1}^{n} \xi_i(\theta_i) - \left(\frac{1}{n-1} \right) \sum_{i=1}^{n} (n-1) \xi_j(\theta_j) \\
\Rightarrow \sum_{i=1}^{n} t_i(\theta) &= 0.
\end{aligned}
$$

The budget balanced payment structure of the agents in the above mechanism can be given a nice graph theoretic interpretation. Imagine a directed graph $G = (V, A)$ where V is the set of $n+1$ vertices, numbered $0, 1, \ldots, n$, and A is the set of $[n + n(n-1)]$ directed arcs. The vertices starting from 1 through n correspond to the n agents involved in the system and the vertex number 0 corresponds to the social planner. The set A consists of two types of the directed arcs:

1. Arcs $0 \rightarrow i \ \ \forall i = 1, \ldots, n$,
2. Arcs $i \rightarrow j \ \ \forall i, j \in \{1, 2, \ldots, n\}\,; i \neq j$.

Each of the arcs $0 \rightarrow i$ carries a flow of $t_i(\theta)$ and each of the arcs $i \rightarrow j$ carries a flow of $\frac{\xi_i(\theta_i)}{n-1}$. Thus the total outflow from a node $i \in \{1, 2, \ldots, n\}$ is $\xi_i(\theta_i)$ and total inflow to the node i from nodes $j \in \{1, 2, \ldots, n\}$ is $-h_i(\theta_{-i}) = \left(\frac{1}{n-1}\right) \sum_{j \neq i} \xi_j(\theta_j)$ Thus for any node i, $t_i(\theta) + h_i(\theta_{-i})$ is the net outflow which it is receiving from node 0 in order to respect the flow conservation constraint. Thus, if $t_i(\cdot)$ is positive then the agent i receives the money from the social planner and if it is negative, then the agent pays the money to the social planner. However, by looking at flow conservation equation for node 0, we can say that total payment received by the planner from the agents and total payment made by the planner to the agents will add up to zero. In graph theoretic terms, the flow from node i to node j can be justified as follows. Each agent i first evaluates the expected total valuation that would be generated together by all his rival agents in his absence, which turns out to be $\xi_i(\theta_i)$. Now, agent i divides it equally among the rival agents and pays to every rival agent an amount equivalent to this. The idea can be better understood with the help of Figure 1, which depicts the three agents case.

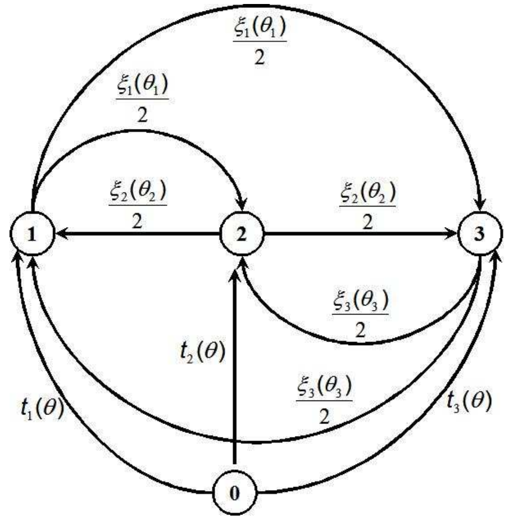

Figure 1: Payment structure showing budget balance in the expected externality mechanism

Example 1 (dAGVA Mechanism for Sealed Bid Auction) Consider a selling agent 0 and two buying agents 1, 2. The buying agents submit sealed bids to buy a single indivisible item. Let θ_1 and θ_2 be the willingness to pay of the buyers. Let us define the usual allocation function:

$$\begin{aligned} y_1(\theta_1, \theta_2) &= 1 \text{ if } \theta_1 \geq \theta_2 \\ &= 0 \text{ else} \\ y_2(\theta_1, \theta_2) &= 1 \text{ if } \theta_1 < \theta_2 \\ &= 0 \text{ else.} \end{aligned}$$

Let $\Theta_1 = \Theta_2 = [0, 1]$ and assume that the bids from the bidders are i.i.d. uniform distributions on [0, 1]. Also assume that $\Theta_0 = \{0\}$. Assuming that the dAGVA mechanism is used, the payments can be computed as follows:

$$t_i(\theta_1, \theta_2) = E_{\theta_{-i}}\left[\sum_{j \neq i} v_j(k(\theta), \theta_j)\right] - \frac{1}{2}\left[\sum_{j \neq i} E_{\theta_{-i}}\left\{\sum_{l \neq j} v_l(k(\theta), \theta_j)\right\}\right].$$

It can be shown that

$$\begin{aligned} t_1(\theta) &= -\left(\frac{1}{12} - \frac{\theta_1}{2} + \frac{\theta_2}{2}\right) y_1(\theta) \\ t_2(\theta) &= -\left(\frac{1}{12} - \frac{\theta_2}{2} + \frac{\theta_1}{2}\right) y_2(\theta) \\ t_0(\theta) &= -(t_1(\theta) + t_2(\theta)) \end{aligned}$$

This can be compared to the first price auction in which case

$$\begin{aligned} t_1(\theta) &= -\frac{\theta_1}{2} y_1(\theta) \\ t_2(\theta) &= -\frac{\theta_2}{2} y_2(\theta). \end{aligned}$$

Also, one can compare with the second price auction, where

$$\begin{aligned} t_1(\theta) &= -\theta_2 y_1(\theta) \\ t_2(\theta) &= -\theta_1 y_2(\theta). \end{aligned}$$

1.3 The Myerson–Satterthwaite Theorem

We have so far not seen a single example where we have all the desired properties in an SCF: AE, BB, BIC, and IR. This provides a motivation to study the feasibility of having all these properties in a social choice function.

The Myerson–Satterthwaite Theorem is a disappointing news in this direction, since it asserts that in a bilateral trade setting, whenever the gains from the trade are possible but not certain, then there is no SCF that satisfies AE, BB, BIC, and Interim IR all together. The precise statement of the theorem is as follows.

Theorem 1.2 (Myerson–Satterthwaite Impossibility Theorem) *Consider a bilateral trade setting in which the buyer and seller are risk neutral, the valuations θ_1 and θ_2 are drawn independently from the intervals $[\underline{\theta_1}, \overline{\theta_1}] \subset \mathbb{R}$ and $[\underline{\theta_2}, \overline{\theta_2}] \subset \mathbb{R}$ with strict positive densities, and $(\underline{\theta_1}, \overline{\theta_1}) \bigcap (\underline{\theta_2}, \overline{\theta_2}) \neq \emptyset$. Then there is no Bayesian incentive compatible social choice function that is ex-post efficient and gives every buyer type and every seller type nonnegative expected gains from participation.*

For a proof of the above theorem, refer to Proposition 23.E.1 of [3].

1.4 Mechanism Design Space in Quasilinear Environment

Figure 2 shows the space of mechanisms taking into account all the results we have studied so far. A careful look at the diagram suggests why designing a mechanism that satisfies a certain combination of properties is quite intricate.

2 Bayesian Incentive Compatibility in Linear Environment

The linear environment is a special, but often-studied, subclass of the quasilinear environment. This environment is a restricted version of the quasilinear environment in the following sense.

1. Each agent i's type lies in an interval $\Theta_i = [\underline{\theta}_i, \overline{\theta_i}] \subset \mathbb{R}$ with $\underline{\theta}_i < \overline{\theta_i}$.
2. Agents' types are statistically independent, that is, the density $\phi(\cdot)$ has the form $\phi_1(\cdot) \times \ldots \times \phi_n(\cdot)$.
3. $\phi_i(\theta_i) > 0 \; \forall \; \theta_i \in [\underline{\theta}_i, \overline{\theta_i}] \; \forall \; i = 1, \ldots, n$.
4. Each agent i's utility function takes the following form

$$u_i(x, \theta_i) = \theta_i v_i(k) + m_i + t_i.$$

The linear environment has very interesting properties in terms of being able to obtain a characterization of the class of BIC social choice functions. Before we present Myerson's Characterization Theorem for BIC social choice functions in a linear environment, we would like to define the following quantities with respect to any social choice function $f(\cdot) = (k(\cdot), t_1(\cdot), \ldots, t_n(\cdot))$ in this environment.

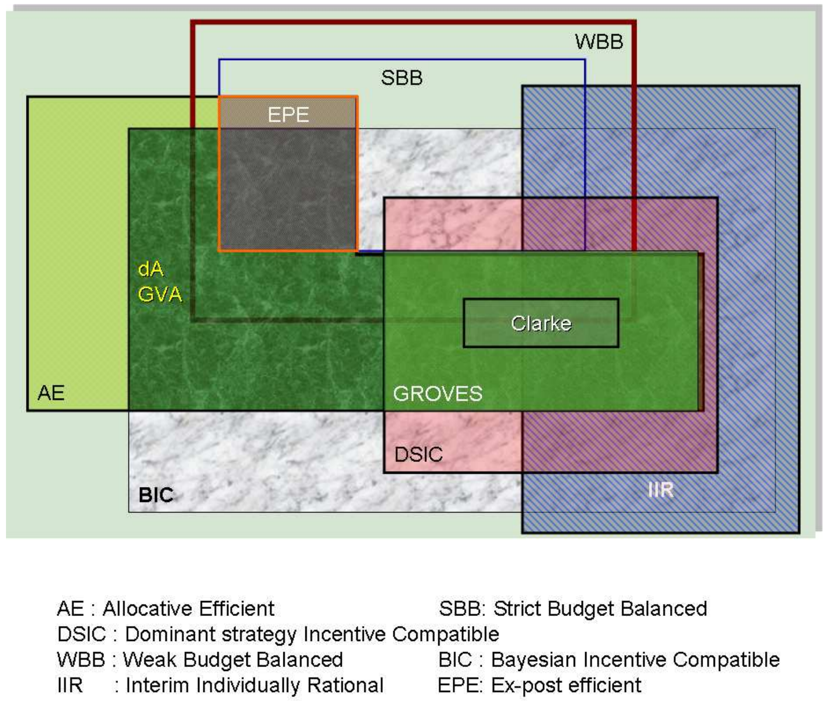

Figure 2: Mechanism design space in quasilinear environment

- Let $\overline{t_i}(\hat{\theta}_i) = E_{\theta_{-i}}[t_i(\hat{\theta}_i, \theta_{-i})]$ be agent i's expected transfer given that he announces his type to be $\hat{\theta}_i$ and that all agents $j \neq i$ truthfully reveal their types.
- Let $\overline{v_i}(\hat{\theta}_i) = E_{\theta_{-i}}[v_i(\hat{\theta}_i, \theta_{-i})]$ be agent i's expected "benefits" given that he announces his type to be $\hat{\theta}_i$ and that all agents $j \neq i$ truthfully reveal their types.
- Let $U_i(\hat{\theta}_i|\theta_i) = E_{\theta_{-i}}[u_i(f(\hat{\theta}_i, \theta_{-i}), \theta_i)|\theta_i]$ be agent i's expected utility when his type is θ_i, he announces his type to be $\hat{\theta}_i$, and that all agents $j \neq i$ truthfully reveal their types. It is easy to verify from the previous two definitions that
$$U_i(\hat{\theta}_i|\theta_i) = \theta_i \overline{v_i}(\hat{\theta}_i) + \overline{t_i}(\hat{\theta}_i).$$
- Let $U_i(\theta_i) = U_i(\theta_i|\theta_i)$ be the agent i's expected utility conditional on his type being θ_i when he and all other agents report their true types. It is easy to verify that
$$U_i(\theta_i) = \theta_i \overline{v_i}(\theta_i) + \overline{t_i}(\theta_i).$$

With the above discussion as a backdrop, we now present Myerson's [4] theorem for characterizing the BIC social choice functions in this environment.

Theorem 2.1 (Myerson's Characterization Theorem) *In linear environment, a social choice function $f(\cdot) = (k(\cdot), t_1(\cdot), \ldots, t_n(\cdot))$ is BIC if and only if, for all $i = 1, \ldots, n$,*

1. *$\overline{v_i}(\cdot)$ is nondecreasing,*

2. $U_i(\theta_i) = U_i(\underline{\theta_i}) + \int_{\underline{\theta_i}}^{\theta_i} \overline{v_i}(s)ds \;\; \forall \, \theta_i$.

For a proof of the above theorem, refer to Proposition 23.D.2 of [3]. The above theorem shows that to identify all BIC social choice functions in a linear environment, we can proceed as follows: First identify which functions $k(\cdot)$ lead every agent i's expected benefit function $\overline{v_i}(\cdot)$ to be nondecreasing. Then, for each such function identify transfer functions $\overline{t_1}(\cdot), \ldots, \overline{t_n}(\cdot)$ that satisfy the second condition of the above proposition. Substituting for $U_i(\cdot)$ in the second condition above, we get that expected transfer functions are precisely those that satisfy, for $i = 1, \ldots, n$,

$$\overline{t_i}(\theta_i) = \overline{t_i}(\underline{\theta_i}) + \underline{\theta_i}\overline{v_i}(\underline{\theta_i}) - \theta_i\overline{v_i}(\theta_i) + \int_{\underline{\theta_i}}^{\theta_i} \overline{v_i}(s)ds$$

for some constant $\overline{t_i}(\underline{\theta_i})$. Finally, choose any set of transfer functions $t_1(\cdot), \ldots, t_n(\cdot)$ such that $E_{\theta_{-i}}[t_i(\theta_i, \theta_{-i})] = \overline{t_i}(\theta_i)$ for all θ_i. In general, there are many such functions, $t_i(\cdot, \cdot)$; one, for example, is simply $t_i(\theta_i, \theta_{-i}) = \overline{t_i}(\theta_i)$.

In what follows we discuss two examples where the environment is linear and analyze the BIC property of the social choice function by means of Myerson's Characterization Theorem.

Example 2 (First-Price Sealed Bid Auction in Linear Environment) Consider the first-price sealed bid auction. Let us assume that $S_i = \Theta_i = \left[\underline{\theta_i}, \overline{\theta_i}\right] \;\; \forall \, i \in N$. In such a case, the first-price auction becomes a direct revelation mechanism $\mathscr{D} = ((\Theta_i)_{i \in N}, f(\cdot))$, where $f(\cdot)$ is an SCF that is the same as the outcome rule of the first-price auction. Let us impose the additional conditions on the environment to make it linear. We assume that

1. Bidders' types are statistically independent, that is, the density $\phi(\cdot)$ has the form $\phi_1(\cdot) \times \ldots \times \phi_n(\cdot)$
2. Let each bidder draw his type from the set $[\underline{\theta_i}, \overline{\theta_i}]$ by means of a uniform distribution, that is $\phi_i(\theta_i) = 1/(\overline{\theta_i} - \underline{\theta_i}) \; \forall \, \theta_i \in \left[\underline{\theta_i}, \overline{\theta_i}\right] \;\; \forall \, i = 1, \ldots, n$.

Note that the utility functions of the agents in this example are given by

$$u_i(f(\theta), \theta_i) = \theta_i y_i(\theta) + t_i(\theta) \; \forall \, i = 1, \ldots, n.$$

Thus, observing $y_i(\theta) = v_i(k(\theta))$ will confirm that these utility functions also satisfy the fourth condition required for a linear environment. Now we can apply Myerson's Characterization Theorem to test the Bayesian incentive compatibility of the SCF involved here. It is easy to see that for any bidder i, we have

$$\begin{aligned} \overline{v_i}(\theta_i) &= E_{\theta_{-i}}[v_i(\theta_i, \theta_{-i})] \\ &= E_{\theta_{-i}}[y_i(\theta_i, \theta_{-i})] \\ &= 1.P\left\{(\theta_{-i})_{(n-1)} \leq \theta_i\right\} + 0.\left(1 - P\left\{\theta_i < (\theta_{-i})_{(n-1)}\right\}\right) \\ &= P\left\{(\theta_{-i})_{(n-1)} \leq \theta_i\right\} \end{aligned} \qquad (2)$$

where $P\left\{(\theta_{-i})_{(n-1)} \leq \theta_i\right\}$ is the probability that the given type θ_i of the bidder i is the highest among all the bidders' types. This implies that in the presence of the independence assumptions made above, $\overline{v_i}(\theta_i)$ is a nondecreasing function.

We know that for a first-price sealed bid auction, $t_i(\theta) = -\theta_i y_i(\theta)$. Therefore, we can claim that for a first-price sealed bid auction, we have

$$\overline{t_i}(\theta_i) \;\; = \;\; -\theta_i\overline{v_i}(\theta_i) \; \forall \, \theta_i \in \Theta_i.$$

The above values of $\overline{v_i}(\theta_i)$ and $\overline{t_i}(\theta_i)$ can be used to compute $U_i(\theta_i)$ in the following manner:

$$U_i(\theta_i) \quad = \quad \theta_i \overline{v_i}(\theta_i) + \overline{t_i}(\theta_i) = 0 \;\; \forall \, \theta_i \in [\underline{\theta_i}, \overline{\theta_i}]. \tag{3}$$

The above equation can be used to test the second condition of Myerson's Theorem, which requires

$$U_i(\theta_i) = U_i(\underline{\theta_i}) + \int_{\underline{\theta_i}}^{\theta_i} \overline{v_i}(s) ds.$$

In view of Equations (2) and (3), it is easy to see that this second condition of Myerson's Characterization Theorem is not being met by the SCF used in the first-price sealed bid auction. Therefore, we can finally claim that a first-price sealed bid auction is not BIC in linear environment.

Example 3 (Second-Price Sealed Bid Auction in a Linear Environment) Consider the second-price sealed bid auction. Let us assume that $S_i = \Theta_i = \left[\underline{\theta_i}, \overline{\theta_i}\right] \quad \forall \, i \in N$. In such a case, the second-price auction becomes a direct revelation mechanism $\mathscr{D} = ((\Theta_i)_{i \in N}, f(\cdot))$, where $f(\cdot)$ is an SCF that is the same as the outcome rule of the second-price auction. We have already seen that this SCF $f(\cdot)$ is DSIC in quasilinear environment, and a linear environment is a special case of a quasilinear environment; therefore, it is DSIC in the linear environment also. Moreover, we know that DSIC implies BIC. Therefore, we can directly claim that the SCF used in the second-price auction is BIC in a linear environment.

3 Problems

1. Consider a sealed bid auction with one seller and two buying agents. There is a single indivisible item which the seller wishes to sell. The bidders are symmetric with independent private values distributed uniformly over $[0, 1]$. Whoever bids higher will be allocated the item. Suppose the dAGVA payment rule is used. Compute the payment that the winner will be required to make. Is the loser also required to make a payment.

2. Consider a bilateral trade setting in which each $\theta_i (i = 1, 2)$ is independently drawn from a uniform distribution on $[0, 1]$. Compute the payments in the dAGVA mechanism. Verify that truth telling is a Bayesian Nash equilibrium.

3. Consider again a bilateral trade setting in which each $\theta_i (i = 1, 2)$ is independently drawn from a uniform distribution on [0,1]. Suppose now that by refusing to participate in the mechanism a seller with valuation θ_1 receives expected utility θ_1 (he simply consumes the good), whereas a buyer with valuation θ_2 receives expected utility 0. Show that in the dAGVA mechanism there is a type of buyer or seller who will strictly prefer not to participate.

4. Consider two agents 1 and 2 with $\Theta_1 = \{1, 2, 3\}$ and $\Theta_2 = \{2, 3\}$. Each of these agents has a single indivisible item to sell to the other. A dAGVA mechanism decides who will sell. Assume that agent 1 sells if his bid is less than or equal to that of agent 2, in which case agent 2 will buy the item from agent 1. If the bid of agent 1 is greater than that of agent 2, then agent 2 sells and agent 1 buys it from agent 2. Payments are decided by the dAGVA mechanism. Design a strictly budget balanced dAGVA mechanism for this problem.

References

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