A Bayesian Incentive Compatible Mechanism for Decentralized Supply Chain Formation

Y. Narahari and Nikesh Kumar Srivastava
Electronics Commerce Laboratory, Computer Science and Automation
Indian Institute of Science, Bangalore, India
hari,nikesh@csa.iisc.ernet.in

Abstract

In this paper, we consider a decentralized supply chain formation problem for linear, multi-echelon supply chains when the managers of the individual echelons are autonomous, rational, and intelligent. At each echelon, there is a choice of service providers and the specific problem we solve is that of determining a cost-optimal mix of service providers so as to achieve a desired level of end-to-end delivery performance. The problem can be broken up into two sub-problems following a mechanism design approach: (1) Design of an incentive compatible mechanism to elicit the true cost functions from the echelon managers; (2) Formulation and solution of an appropriate optimization problem using the true cost information. In this paper, we propose a novel Bayesian incentive compatible mechanism for eliciting the true cost functions. This improves upon existing solutions in the literature which are all based on the classical Vickrey-Clarke-Groves mechanisms, requiring significant incentives to be paid to the echelon managers for achieving dominant strategy incentive compatibility. The proposed solution, which we call SCF-BIC (Supply Chain Formation with Bayesian Incentive Compatibility), significantly reduces the cost of supply chain formation. We illustrate the efficacy of the proposed methodology using the example of a three echelon manufacturing supply chain.

Keywords

Supply chain planner (central design authority) (CDA), mean variance allocation, mechanism design, Groves mechanism, dAGVA (d'Aspremont and Gérard-Verat) mechanism, SCF-DSIC (Supply Chain Formation-Dominant Strategy Incentive Compatible), SCF-BIC (Supply Chain Formation-Bayesian Incentive Compatible).

1 Introduction

In this paper, we describe a decentralized supply chain formation problem where the supply chain planner or a central design authority (CDA) is faced with the decision of choosing a partner or service provider at individual supply chain echelons so as to meet delivery quality levels at minimum cost. If all the relevant information is available in accurate form with the CDA, then the problem could be formulated and solved as an appropriate optimization problem. In such a case, the problem becomes a centralized one. However, in the real-world, the CDA does not always have access to all the required information. The primary reason for this is the fact that typical supply chain entities (such as echelon managers and service providers) are autonomous, rational, and intelligent, and consequently exhibit strategic behavior. These entities may not reveal their true cost information; in fact, they may provide false information in the best hope of maximizing their individual utility functions. Eliciting truthful information is key to forming an optimal supply chain but calls for payment of significant incentives to the rational entities. Due to this, the supply chain formation problem becomes a decentralized one. In this paper, we wish to solve such supply chain formation problems with *incomplete information* in a cost-effective way.

The specific problem we solve is that of choosing an optimal mix of supply chain partners or service providers in a linear multi-echelon supply chain so as to achieve a prescribed level of delivery performance in the presence of autonomous, rational, and intelligent echelon managers. We now briefly review the relevant literature to set the context for the paper.

1.1 Review of Relevant Work

We can classify relevant literature on supply chain formation problem into two categories. The first category includes papers which make an implicit assumption of complete information, that is the supply chain planner or decision maker has access to all the relevant information required to solve the problem. The second category includes papers which assume incomplete information, that is, the supply chain planner does not have access to all the information. Our contribution falls into the second category and our literature review therefore is restricted to this case. The approach to address the supply chain formation problem with incomplete information has followed two tracks: the first uses techniques based on competitive equilibrium analysis [14, 13] and the second makes use of auction technology [3, 15, 12, 4, 9].

1.1.1 Competitive Equilibrium Models

In the competitive equilibrium approach of [14, 13], the authors address the supply chain formation problem from the perspective

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of a third-party market maker. This market maker is interested in constructing the supply chains of multiple end customers given the fact that they could potentially share some common suppliers. They construct a network of possible supply chain configurations and use a price directed search for a feasible supply chain configuration, that is, one that maintains material balance and profitability. The price directed search leads to what is essentially an approximate competitive equilibrium which is in line with standard results in competitive equilibrium theory for indivisible goods. They describe two distributed protocols to find the equilibrium prices along with the supply chain configuration.

1.1.2 Auction Based Models

In [3], the authors consider a linear supply chain where a commodity market exists for each of the goods - both final and intermediate, that can be traded. The supply chain formation protocols that are discussed build on double auction rules where buyers and sellers submit asks and bids simultaneously. The protocol as such governs the construction of supply and demand curves at each of the markets (supply, intermediate, and final). In each market, after the supply and demand curves are constructed, a double auction rule is invoked to finalize the traded quantities and the prices. When the double auction rule is based on the Vickrey-Clarke-Groves (VCG) payment rule, the supply chain formation protocol turns out to be strategy-proof, efficient, individually rational but not budget balanced. This idea is further extended in [4] to more general supply chain structures where two or more goods maybe used for making a single good.

Another approach to modeling the supply chain formation problem has been to use combinatorial auction/exchange technology; some of the problems related to uncoordinated action across the supply chain are avoided by this. In [15], the supply chain formation problem is modeled for single minded supply chain agents (buyers and sellers), where single minded refers to the the fact that the agents are interested in one particular bundle of goods alone. The agents submit bids for bundles of goods to a central auction/exchange. The auction/exchange then solves a combinatorial optimization problem and indicates the allocations.

In [9], the supply chain formation problem is modeled as a multi-commodity flow problem which can be solved using standard LP decomposition techniques. Since the cost functions are under the control of agents who solve the decomposed problems, the authors argue that a more natural way to solve the problem is to use a combinatorial auction based protocol. The combinatorial auction is constructed to minimize the cost of forming the supply chain while meeting the demands of the customer.

The paper by Garg *et al* [7] describes a category of supply chain formation problems where the supply chain planner is faced with the decision of choosing a partner or service provider for each echelon of the supply chain so as to achieve required delivery quality levels at minimum cost. The delivery quality is expressed in terms of minimum levels of supply chain process capability indices C_p, C_{pk}, and C_{pm}. The authors show that the VCG mechanisms could be used to elicit true costs from the echelon managers. They illustrate their approach with the help of an example of forming a three stage distribution process of a typical automotive supply chain. The optimization problem is this case is a *variance pool allocation* problem which allocates an optimal variance to individual stages of the supply chain based on the cost functions supplied by the alternate service providers.

1.2 Contributions and Outline

All existing mechanism design based approaches for the supply chain formation problem are based on the classical VCG mechanisms [1, 6]. This guarantees dominant strategy incentive compatibility (DSIC) which implies that each echelon manager finds it a best response to reveal his/her true cost function irrespective of what the other echelon managers report. This is an extremely desirable property; however, to satisfy such a strong property, it turns out that the incentive payments to the echelon managers could be prohibitively high. Motivated by this, in this current paper, we extend the existing work by weakening the incentive compatibility to that of Bayesian incentive compatibility (BIC) which only guarantees that it is optimal for an echelon manager to report the truth whenever the other echelon managers are also truthful. We show that this weaker property ensures that the incentive payments to the echelon managers are much less; consequently the desired level of delivery performance is achieved at lower cost. To the best of our knowledge, this is the first effort in the direction of exploring Bayesian incentive compatible mechanisms for the supply chain formation problem.

There is one additional direction in which the current paper extends the existing state-of-the-art. The optimization problem in Garg *et al* [7] is a variance pool allocation problem where an optimal lead time variance is chosen for each echelon of the supply chain so as to achieve the required delivery performance at minimum cost. This has the limitation that the mean of the lead time that can be quoted by the alternate service providers at any given echelon is fixed. We overcome this limitation by extending the formulation to that of a mean-variance allocation (MVA) approach. Now, the service providers at each echelon would be able to report their costs as a function of both mean and variance.

The rest of this paper is organized as follows. In Section II, we first assume that the cost functions are available in truthful form from the echelon managers and formulate an optimization problem that describes the supply chain formation problem (with complete information). In Section III, we take up the supply chain formation problem with incomplete information and set up the mechanism design problem. Section IV is devoted to developing a Bayesian incentive compatible solution (SCF-BIC) to the problem. In Section V, we discuss a case study of forming a manufacturing supply chain with three echelons: Casting, Machining, and Transportation. We present appropriate numerical experiments to highlight the superior performance of the proposed mechanism.

2 Supply Chain Formation: A Formulation based on Mean-Variance Allocation

Let us consider an n-echelon linear supply chain with stochastic lead times. See Figure 1. Let us assume the following.

1. At each echelon of the supply chain, there exist multiple service providers in the market who can offer the service

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required at that echelon. The time taken by each service provider to deliver a unit order is stochastic in nature and is normally distributed.

2. The candidate service providers at echelon i may offer different values of mean, standard deviation, and delivery cost for a unit order. The delivery cost is a function of the mean and the standard deviation. The service providers provide this information truthfully to their echelon manager. The manager of echelon i uses this information to come up with an aggregate cost function $\nu_i(\mu_i, \sigma_i)$ for echelon i, where μ_i is the mean and σ_i is the standard deviation. This aggregate cost function can be obtained, for example, by performing a polynomial curve fitting using the costs reported by the various service providers for echelon i. Such a cost function $\nu_i(\mu_i, \sigma_i)$ will capture the cost versus delivery performance trade-offs across different service providers available for echelon i.

3. The delivery times $X_i, \quad \forall i = 1, \ldots, n$ of the various echelons in the n-echelon supply chain are independent random variables and there is no time elapsed between the end of process i and commencement of process $i+1, \forall i = 1, ..., n-1$. As an immediate consequence, the end-to-end delivery time Y of an order is

$$Y = \sum_{i=1}^{n} X_n$$

It can noted immediately that Y is normally distributed with $\mu = \sum_{i=1}^{n} \mu_i$ and variance $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2$.

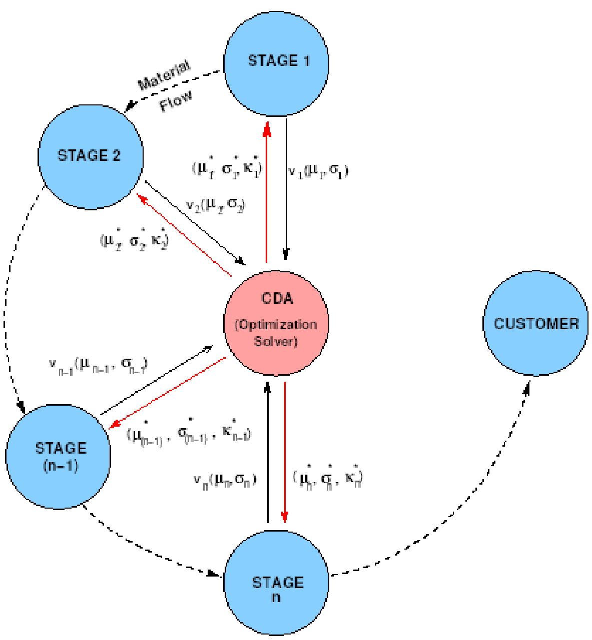

Figure 1. The supply chain formation problem for a linear, multi-echelon supply chain

2.1 Mean Variance Allocation Problem

We assume that the CDA's target is to deliver the orders to the customers within $\tau \pm T$ days of receiving the order. We call τ as the delivery target date and T as the tolerance. We also define $L = \tau - T$ to be the lower limit of the delivery window and $U = \tau - T$ be the upper limit of the delivery window. The CDA measures the delivery performance of the supply chain in terms of how precisely and accurately an order is delivered to the customer within the delivery window $(\tau \pm T)$. The problem of the CDA is to minimize the end-to-end delivery cost subject to delivery performance constraints. We formulate this optimization problem in terms of the *supply chain process capability indices* C_p, C_{pk}, and C_{pm} [8]. The three indices C_p, C_{pk}, and C_{pm} for the end-to-end delivery time Y are defined in the following manner [8]:

$$C_p = \frac{U-L}{6\sigma} = \frac{T}{3\sigma} \quad (1)$$

$$C_{pk} = \frac{\min(U-\mu, \mu-L)}{3\sigma} \quad (2)$$

$$C_{pm} = \frac{T}{3\sqrt{\sigma^2+b^2}} \quad (3)$$

where $b = |\tau - \mu|$. The following relations among these three indices can be easily verified [8].

$$C_p \geq C_{pk} \geq 0 \quad (4)$$

$$C_p \geq C_{pm} \geq 0 \quad (5)$$

$$C_{pk} = C_p(1-k) \text{ where } k == \frac{|\tau-\mu|}{T} \quad (6)$$

$$\frac{1}{9C_{pm}^2} = \frac{1}{9C_p^2} + \left(1 - \frac{C_{pk}}{C_p}\right)^2 \quad (7)$$

The probability of delivering an order within a specified delivery window $\tau \pm T$, which is known as *delivery probability*, of the end-to-end delivery time Y can be expressed in terms of its capability indices C_p and C_{pk} in the following way [8].

$$\text{DP} = \Phi(3C_{pk}) + \Phi(6C_p - 3C_{pk}) - 1 \quad (8)$$

where $\Phi(.)$ is the cumulative distribution function of standard normal distribution. The index C_{pm} measures how closely the orders are being delivered to the target delivery date and is known as *delivery sharpness* of the end-to-end delivery time Y [8]. The CDA first invites each echelon manager to submit its cost function $\nu_i(\mu_i, \sigma_i)$. Assuming that true cost functions are elicited from each echelon, the CDA just needs to solve an optimization problem that will minimize the expected end-to-end delivery cost while ensuring certain specified levels of delivery performance. The solution of the optimization problem results in optimal values of the design parameters (μ_i^*, σ_i^*) which are communicated back to the respective echelon managers (see Figure 1) by the CDA. The CDA also allocates budget $\kappa_i = \nu_i(\mu_i, \sigma_i)$ for the manager of echelon $i \quad \forall i = 1, 2, ..., n$. The following parameters are known to the CDA.

1. The delivery window $(\tau - T, \tau + T)$.

2. Lower bounds on the values of C_p and C_{pk}, say for example $C_p \geq p$ and $C_{pk} \geq q$. Note that it is enough to specify C_p and C_{pk} since the value of C_{pm} is automatically decided by C_p and C_{pk}.

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3. Delivery cost function $\nu_i(\mu_i, \sigma_i)$ per unit order submitted by the manager of echelon i.

The decision variables are the means and the standard deviations of each individual echelon $i \quad (i = 1, ..., n)$. The objective function is the end-to-end delivery cost. Thus, the optimization problem to be solved is:

$$\text{minimize} \quad \sum_{i=1}^{n} \nu_i(\mu_i, \sigma_i) \tag{9}$$

subject to:

$$C_p \geq p \tag{10}$$

$$C_{pk} \geq q \tag{11}$$

$$\tau - T \leq \sum_{1}^{n} \mu_i \leq \tau + T \tag{12}$$

$$\mu_i \geq 0, \sigma_i \geq 0 \quad \forall i \tag{13}$$

An interesting approach to the above optimization problem is to solve it as what we can call the *Mean Variance Allocation* (MVA) problem. It is reasonable to assume that the cost function at stage i ($i = 1, 2, \ldots, n$ depends on μ_i and σ_i. In particular, we assume a cost function $\nu_i(\mu_i, \sigma_i)$ of the following form:

$$\nu_i(\mu_i, \sigma_i) = \{a_{i0} + a_{i1}\mu_i + a_{i2}\sigma_i + a_{i3}\mu_i\sigma_i + a_{i4}\sigma_i^2\} \tag{14}$$

The manager of each echelon submits a 5-tuple $(a_{i0}, a_{i1}, a_{i2}, a_{i3}, a_{i4})$ to the CDA. Thus, the problem of the CDA is to use the above cost functions reported by the echelon managers and solve the optimization problem (9) - (13) to obtain an optimal mean μ_i^* and an optimal standard deviation σ_i^* for each echelon i.

3 A Mechanism Design Approach to Supply Chain Formation

The problem formulation discussed in the previous section is based on the critical assumption that the echelon managers submit the true cost functions to the CDA. This assumption is rarely true in a real-world situation since the managers of the echelons are rational, intelligent, and autonomous. Therefore, it is not surprising if the manager of each echelon reports untruthful cost function to the CDA (because in their perception, doing so may help them improve their own individual utilities). On the other hand, the CDA typically sets forth an overall system wide goal and wants all the echelon managers to plan and operate in a way that is aligned with this system wide goal. In this setting, the problem of the CDA is to design an incentive compatible mechanism in order to elicit the true *types* of the echelon managers. Here, the true type of echelon manager corresponds to the true cost function which is the private information of the manager.

We assume that $\nu_i(.)$ is the actual cost function of echelon i, which is known only to the manager of echelon i. On receiving a request from the CDA, the i^{th} echelon manager reports a cost function $w_i(.)$ to the CDA which need not be the same as $\nu_i(.)$.

3.1 Setup for Mechanism Design

The underlying game, which we call the supply chain formation game, is a strategic form game with incomplete information (in particular, a Bayesian game). The following are some key components of this game.

- C_i=Set of possible values for mean μ_i
- D_i=Set of possible values for standard deviation σ_i
- $\mu = (\mu_1, \mu_2, ..., \mu_n)$, a profile of mean delivery times at the echelons
- $\sigma = (\sigma_1, \sigma_2, ..., \sigma_n)$, a profile of standard deviations delivery times at the echelons
- $\nu_i : C_i \times D_i \to \mathbb{R}$=True cost function (actual type) of manager of echelon i
- $w_i : C_i \times D_i \to \mathbb{R}$=Reported cost function (reported type) of manager of echelon i
- V_i= Set of all possible types of manager of echelon i
- W_i=Set of all possible reported types of manager of echelon i.

We make the following specific assumptions regarding actual types and reported types of the echelon manager.

1. The μ_i can take values from the set $(0, \overline{\mu_i}]$, that is C_i=$(0, \overline{\mu_i}]$
2. The σ_i can take values from the set $(0, \overline{\sigma_i}]$, that is D_i=$(0, \overline{\sigma_i}]$
3. The actual type of manager of echelon i is of the following form
   $\nu_i(\mu_i, \sigma_i)$=$a_{i0} + a_{i1}\mu_i + a_{i2}\sigma_i + a_{i3}\mu_i\sigma_i + a_{i4}\sigma_i^2 \quad \forall \sigma_i \in (0, \overline{\sigma_i}]$ and $\forall \mu_i \in (0, \overline{\mu_i}]$
4. For each echelon i, it is possible to obtain an interval for each one of the all the coefficient that is
   $a_{i0} \in [\underline{a_{i0}}, \overline{a_{i0}}]; a_{i1} \in [\underline{a_{i1}}, \overline{a_{i1}}]; a_{i2} \in [\underline{a_{i2}}, \overline{a_{i2}}]; a_{i3} \in [\underline{a_{i3}}, \overline{a_{i3}}]; a_{i4} \in [\underline{a_{i4}}, \overline{a_{i4}}]$ These intervals are such that choosing the coefficient from them will always result in cost function $\nu_i(\sigma_i, \mu_i)$ that is non-negative and non-increasing.

The general structure of social choice function for this case is

$$f(w) = ((\mu_i(w), \sigma_i(w)), I_i(w))_{i=1,\ldots,n} \quad \forall w \in V \tag{15}$$

where $I_i(w)$ is the payment received by agent i. The CDA wants to achieve a desired level of delivery quality at minimum cost, so the social choice function that the CDA wishes to implement is the following:

$$f(w) = ((\mu_i^*(w), \sigma_i^*(w)), I_i(w))_{i=1,\ldots,n} \quad \forall w \in V \tag{16}$$

where $(\mu i^*(w))_{i=1,\ldots,n}$ and $(\sigma_i^*(w))_{i=1,\ldots,n}$ provides the solution of the optimization problem faced by the CDA.

In view of the above definition of $(\sigma_i^*(w), \mu_i^*(w))_{i=1,\ldots,n}$, it is easy to see that social choice function is allocatively efficient. Now if the payments $(I_i(w))_{i=1,\ldots,n}$ are chosen properly, then it is possible that the social choice function becomes incentive compatible in which case no manager will have incentive to report untruthful cost function. Thus the problem of the CDA will we solved if we come up with an allocatively efficient and incentive compatible social choice function.

An outcome x is a vector $((\mu_i, \sigma_i), I_i)_{i=1,\ldots,n}$ where

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- σ_i is the allowed variability in delivery time at echelon i and μ_i is mean delivery time at each echelon i. This means the manager for echelon i needs to choose a service provider who can offer the service with variability and mean less than equal or equal to σ_i and μ_i respectively.
- I_i is the total budget sanctioned by the CDA for the manager of echelon i. The manager is supposed to procure the service for echelon i within this budget. If there is a surplus with this manager, then that is the utility or payoff for the manager and the department. On the other hand, if there is any deficit, then the manager needs to mobilize the additional funds from his/her own department.

In such a case, the set of feasible outcomes is

$$X = \{((\mu_i, \sigma_i), I_i)_{i=1,\dots,n} | \mu_i \in C_i, \sigma_i \in D_i, \text{ and } I_i \in \mathbb{R}\} \tag{17}$$

The utility function of a manager for echelon i is given for x=$((\mu_i, \sigma_i), I_i)_{i=1,\dots,n}$, by

$$u_i(x, \nu_i) = I_i - \nu_i(\mu_i, \sigma_i) \tag{18}$$

3.2 Dominant Strategy Incentive Compatible Supply Chain Formation (SCF-DSIC)

The VCG payment I_i for echelon i is given by the following relation [1, 6].

$$I_i(w) = \alpha_i - \sum_{j \neq i} w_j(\mu_i^*(w), \sigma_i^*(w)) \quad \forall w \in V \tag{19}$$

where $(\mu_1^*(w), ..., \mu_n^*(w))$ and $(\sigma_1^*(w), ..., \sigma_n^*(w))$ are the optimal solutions of optimization problem faced by the CDA . Here α_i is some constant which can be assumed to be the same for all the echelons and can be used to normalize the value of I_i so that it has some meaningful value. We can say that if the CDA directly asks the echelon managers to report their cost function and uses the reported cost function $w_1, ..., w_n$ in order to compute

- the mean delivery times $(\mu_i^*(w))_{i=1,\dots,n}$ and standard deviations of delivery times $(\sigma_i^*(w))_{i=1,\dots,n}$ by solving the optimization problem (9)-(13), and
- the payments $I_i(w)$ by using the VCG payment scheme given in (19),

then, by the Groves Theorem [1, 6], the above mechanism becomes dominant strategy incentive compatible, that is, all echelon managers will find it their best response to report their true cost functions irrespective of what the other echelon managers report.

In summary, we are able to come up with a supply chain formation that induces truth revelation by all the agents in a dominant strategy sense. Because of the stringent requirement of dominant incentive compatibility, the payments to be made to the echelon managers are bound to be quite high. By weakening the notion of incentive compatibility to that of Bayesian incentive compatibility, we can form the supply chain at a lower cost. We discuss this next.

4 Bayesian Incentive Compatible Supply Chain Formation (SCF-BIC)

Let $k^*(w) = (\mu_1^*(w), \dots, \mu_n^*(w), \sigma_1^*(w), \dots, \sigma_n^*(w))$ be an allocation that is allocatively efficient. That is, it is a solution of the optimization problem (9-13). Let $I_1(w), \dots, I_n(w)$ denote the payments to be received by the players $1, \dots, n$ respectively. We now state the dAGVA theorem (d'Aspremont and Gérard-Varet [5] and Arrow [2]) for this setting.

The dAGVA Theorem

Let $f(.) = (k^*(.), I_1(.), \dots, I_n(.))$ be allocatively efficient and the agents types be statistically independent of one another. Then $f(.)$ is Bayesian incentive compatible under the following payment structure.

$$\begin{aligned} &\forall i \in N && \forall \theta \in V \\ &I_i(\theta_i, \theta_{-i}) = \beta_i(\theta_{-i}) && +E_{\tilde{\theta}_{-i}}[\sum_{j \neq i} v_j(k^*(\theta_i, \tilde{\theta}_{-i}), \theta_j)] \end{aligned}$$

where $\beta_i : V_i \to R$ is any arbitrary function. Moreover, it is always possible to choose the functions $\beta_i(.)$ such that

$$\sum_{i=1}^{n} I_i(\theta) = 0 \ \ \forall \theta \in v$$

The statistical independence assumption above means that the belief distributions $p_i : V_i \to \Delta V_i$ of the individual agents are mutually independent. That is, each agent's beliefs about the types of the other agents are independent of independent of the other agents' belief distributions.

The expectation term

$$E_{\tilde{\theta}_{-i}}[\sum_{j \neq i} v_j(k^*(\theta_i, \tilde{\theta}_{-i}), \theta_j)]$$

represents the expected payment for an agent i when the agent i announces its type to be θ_{-i} and all other agents $j \neq i$ report their true types. The functions $\beta_i(.)$ can be chosen so as to guarantee budget balance as shown in [1, 10]. Define

$$\xi_i(\theta_i) = E_{\tilde{\theta}_{-i}}[\sum_{j \neq i} v_j(k^*(\theta_i, \tilde{\theta}_{-i}), \theta_j)]$$

and

$$\beta_i(\theta_{-i}) = -(\frac{1}{n-1}) \sum_{j \neq i} \xi_j(\theta_j) \ \ \forall i \in N$$

Then it can be easily shown that

$$\sum_{i=0}^{n} I_i(\theta) = 0$$

The expectation term for agent $j \neq i$ in the expression represents the *expected payment* for agent j when the agent i announces its type to be w_i and agent j reports the truth. Let q_{ij} be the probability that agent i (that is, echelon manager i) believes that agent j is truthful when agent i is truthful and q'_{ij} the probability that agent i believes that agent j is untruthful when agent i is truthful. Then

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we note immediately that $q_{ij} + q_{ij}^{'} = 1 \quad \forall i \neq j$. The payment to echelon manager i with belief probabilities q_{ij} $(j = 1, \ldots, n)$ is given by:

$$I_i = \sum_{j \neq i}^{n} q_{ij} w_j(\mu_j, \sigma_j) \tag{20}$$

In summary, we are able to form a supply chain which guarantees allocative efficiency, budget balance, and Bayesian incentive compatibility. We now show, using a case study, that SCF-BIC can be achieved at a cost that is lower than that of SCF-DSIC.

5 A Case Study

We consider the problem of forming a three echelon supply chain network as shown in Figure 2. We make the following specific assumptions in this case study.

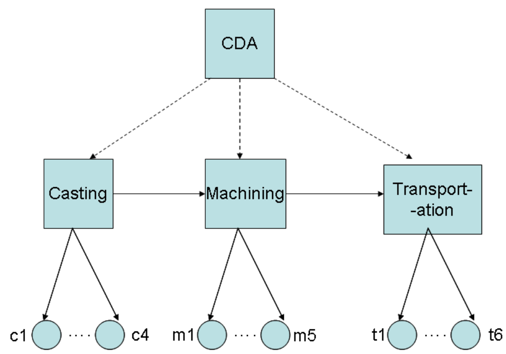

Figure 2. A Three echelon supply chain

1. There are three echelons in supply chain: (1) casting stage with 4 service providers or partners; (2) machining stage with 5 partners, and (3) transportation stage with 6 partners.

2. For each echelon, the mean delivery time, the standard deviation of delivery time, and the cost vary across alternate service providers. Tables 1, 2, and 3 show these values. We assume that the service providers truthfully reveal these values to the respective echelon managers.

3. The manager for each echelon i $(i = 1, 2, 3)$ uses his/her private information to compute the true cost function by fitting the following polynomial:
$\nu_i(\mu_i, \sigma_i)$=$a_{i0} + a_{i1}\mu_i + a_{i2}\sigma_i + a_{i3}\mu_i\sigma_i + a_{i4}\sigma_i^2$.
The coefficients $a_{i0}, a_{i1}, a_{i2}, a_{i3}$, and a_{i4} constitute the private information of the echelon managers.

4. The CDA has an ideal target of an end to end delivery time of 6 days with a tolerance of 2 days. Thus the delivery window is $(4, 8)$ days.

5. The CDA wants to choose the service provider for each echelon in a way that the values $C_p^* \geq 1.8$ and $C_{pk}^* \geq 1.08$ are attained for the end-to-end delivery time Y. Note that we have taken some reasonable but arbitrary thresholds for C_p and C_{pk}.

Partner Id	μ (days)	σ (days)	Cost
P_{11}	3	1.0	105
P_{12}	3	1.5	70
P_{13}	2	0.5	55
P_{14}	2	1.0	40

Table 1. True types revealed by the service providers to the manager of the casting stage (Echelon 1)

Partner Id	μ (days)	σ (days)	Cost
P_{21}	3	0.75	35
P_{22}	2	1.00	40
P_{23}	2	1.25	35
P_{24}	2	0.75	50
P_{25}	1	1.00	70

Table 2. True types revealed by the service providers to the manager of the machining stage (Echelon 2)

Partner Id	μ (days)	σ (days)	Cost
P_{31}	1	0.25	20
P_{32}	1	0.5	15
P_{33}	2	0.75	12
P_{34}	2	1.00	10
P_{35}	2	1.25	9
P_{36}	2	1.50	8

Table 3. True types revealed by the service providers to the manager of the transportation stage (Echelon 3)

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Echelon i	μ_i^* days	σ_i^* days	$\nu_i(\mu_i^*, \sigma_i^*)$
1	2	0.2613	161.56
2	1	0.0149	72.63
3	2	0.01	24.0

Table 4. Optimal values of mean and standard deviation and optimal costs as computed by the supply chain manager

Echelon i	Payments for SCF-DSIC	Payments for SCF-BIC
1	253.4	21
2	164.4	16.53
3	115.8	14.11

Table 5. Payments using SCF-DSIC and SCF-BIC in Case 1

Echelon i	Payments for SCF-DSIC	Payments for SCF-BIC
1	253.4	105
2	164.4	82.65
3	115.8	70.56

Table 6. Payments using SCF-DSIC and SCF-BIC in Case 2

Echelon i	Payments for SCF-DSIC	Payments for SCF-BIC
1	253.4	188.9
2	164.4	148.9
3	115.8	127

Table 7. Payments using SCF-DSIC and SCF-BIC in Case 3

As discussed earlier, in the complete information case, each echelon manger submits the true cost function and the CDA just solves the resulting optimization problem. Solving the optimization problem (9-13) for this case will yield the values shown in Table 4. A Lagrange multiplier method has been used to solve this problem. Now we compute the payments to the echelon managers when the SCF-DSIC and SCF-BIC mechanisms are used. For the case of SCF-DSIC, we assume $\alpha_1 = \alpha_2 = \alpha_3$=350. In the case of SCF-BIC, we consider four representative cases as shown below:

- Case 1: $q_{12} = q_{13} = q_{21} = q_{23} = q_{31} = q_{32} = 0.1$. This case corresponds to a situation where each agent believes that the other agents are highly untruthful.
- Case 2: $q_{12} = q_{13} = q_{21} = q_{23} = q_{31} = q_{32} = 0.5$. This case corresponds to a situation where each agent believes that the other agents are equally likely to be truthful or untruthful.
- Case 3: $q_{12} = q_{13} = q_{21} = q_{23} = q_{31} = q_{32} = 0.9$. This case is a a situation where each agent believes that the other agents are highly truthful.
- Case 4: $q_{12} = q_{13} = q_{21} = q_{23} = q_{31} = q_{32} = 1$. Here each agent believes that the other agents are completely truthful.

Tables 5, 6, 7, and 8 compare the payments to the echelon managers in Cases 1, 2, 3, and 4, respectively. It is clear that SCF-BIC achieves the required delivery performance at much lower cost. Also, note that the payments to the echelon managers with the SCF-BIC mechanism progressively increase with increase in the values of the belief probabilities. This confirms the fact that the echelon managers can expect to get higher payments as they become progressively more truthful due to the corresponding increase in the expected externalities on the system.

6 Summary and Future Research

In this paper, we considered a decentralized supply chain formation problem for linear, multi-echelon supply chains when the managers of the individual echelons are autonomous, rational, and intelligent. We proposed a novel Bayesian incentive compatible mechanism for eliciting the true cost functions from the echelon managers. This improves upon existing solutions in the literature which are all based on the classical Vickrey-Clarke-Groves mechanisms and which require significant incentives to be paid to the echelon managers for achieving dominant strategy incentive compatibility. In the proposed solution, incentive compatibility is achieved at much lower cost thus significantly reducing the cost of supply chain formation.

6.1 Future Research

One can investigate *optimal mechanisms* for this problem on the lines of Myerson's optimal auction design [11]. Such a mechanism will lead to minimum cost supply chain formation subject to Bayesian incentive compatibility and interim individual rationality.

In practice, the echelon managers may not be intelligent enough to understand the idea behind incentive compatible mechanisms. A manager-friendly approach would be design an iterative scheme which will help the stage managers learn that truth revelation is an optimal strategy.

In this paper, we have modeled only the echelon managers as rational and intelligent. We have implicitly assumed that the indi-

Echelon i	Payments for SCF-DSIC	Payments for SCF-BIC
1	253.4	209.9
2	164.4	165.41
3	115.8	141.1

Table 8. Payments using SCF-DSIC and SCF-BIC in Case 4

vidual service providers at each echelon are truthful. The approach followed in this paper can be extended to take care of strategic behavior by service providers also.

The formulation in this paper assumes linear supply chains. The case of non-linear supply chains can be addressed by suitably extending the optimization problem formulations.

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