Proceedings of the 3rd Annual
IEEE Conference on Automation Science and Engineering
Scottsdale, AZ, USA, Sept 22-25, 2007

A Shapley Value Analysis to Coordinate the Formation of Procurement Networks

T S Chandrashekar and Y Narahari

Abstract— **In this paper we address the problem of forming procurement networks for items with value adding stages that are linearly arranged. Formation of such procurement networks involves a bottom-up assembly of complex production, assembly, and exchange relationships through supplier selection and contracting decisions. Recent research in supply chain management has emphasized that such decisions need to take into account the fact that suppliers and buyers are intelligent and rational agents who act strategically. In this paper, we view the problem of Procurement Network Formation (PNF) for multiple units of a single item as a cooperative game where agents cooperate to form a surplus maximizing procurement network and then share the surplus in a fair manner. We study the implications of using the Shapley value as a solution concept for forming such procurement networks. We also present a protocol, based on the extensive form game realization of the Shapley value, for forming these networks.**

I. INTRODUCTION

Consider a stylised supply chain for an automotive stamping which typically spans many tiers. An automotive assembler, hereafter called the buyer, is interested in procuring stampings for assembly. The buyer values the item at a certain price. The stamping undergoes many processes before it can be delivered to the buyer. Starting from the master coil, it undergoes cold rolling, pickling, slitting, and stamping. We assume that all these manufacturing operations are organized linearly and precedence constraints apply as shown in Figure 1 to the way in which the operations can be carried out.

Now, a wide variety of suppliers with varying capabilities may be available. Each of these suppliers incurs costs to carry out the processing at various stages of value addition. It is safe to assume that these costs vary from firm to firm and these costs may be commonly known or may be privately held information.

Given all the different options in which the stamping can be procured, the buyer has to now decide what is the best (least cost) combination of suppliers he can put together to carry out the various operations. This can be done by constructing what we call the *procurement feasibility graph* as shown in Figure 2.

The authors would like to acknowledge the support provided by General Motors Research and Development, Warren, MI, USA and General Motors, India Science Lab, Bangalore, India for this research.

This work was carried out when Chandrashekar was a Ph.D student at the Department of Computer Science and Automation, Indian Institute of Science, Bangalore, India. He can be reached at chandra@csa.iisc.ernet.in

Y. Narahari is with the Department of Computer Science and Automation, Indian Institute of Science, Bangalore, India hari@csa.iisc.ernet.in

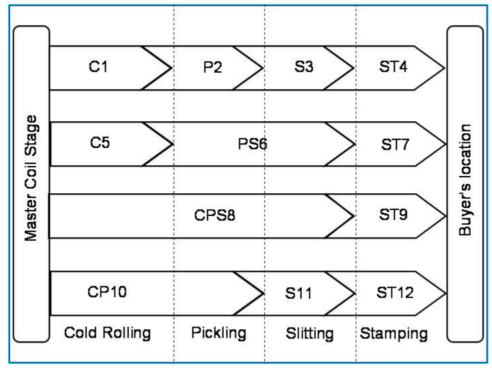

Fig. 1. A linear supply chain for automotive stampings

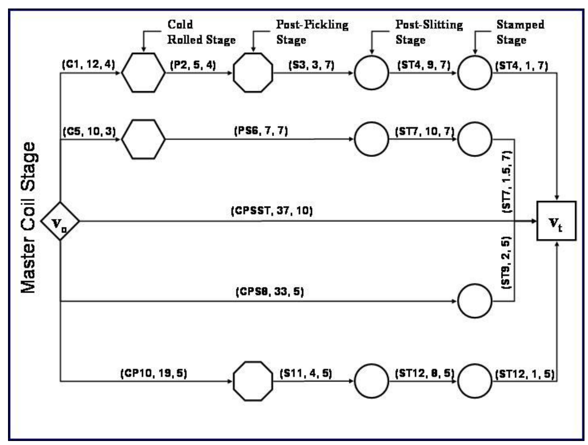

Fig. 2. A procurement feasibility graph

In this graph, each edge represents one of the value adding operations. An edge is assumed to be owned by a supplier, hereafter called an agent. The fact that each agent incurs a certain cost of processing is captured as the cost of allowing a unit amount of flow on the edge owned by the agent. Also, the processing capacity of the supplier for a particular operation is indicated by an upper bound on the flow that is possible along the edge representing that supplier's operation. In Figure 2, we capture this information as a 3-tuple beside every edge in the graph. Further, we assume that there are two special nodes in the graph called the origin node and the terminal node that represent the starting point and ending point respectively for all value adding operations.

A path from the origin node to the terminal node in the graph indicates one possible way of procuring one unit of the stamping. The cost of this procurement is obtained by

1-4244-1154-8/07/$25.00 ©2007 IEEE.

summing the costs of the individual edges in that path. More generally, a flow of f units along a path costs f times the cost of that path. A buyer is then naturally interested in finding a minimum cost way of procuring the stampings to meet his demand or in other words finding a minimum cost flow through the network that meets his demand.

Consider now a scenario where an independent market maker wishes to match the buyer's demand with the suppliers' capabilities. The objective of such a market maker would be to maximize the surplus value that could be generated by matching suppliers with the buyer. Here we define surplus value as the difference between buyer's valuation times the amount of flow through the network and the cost of maintaining that flow. This is nothing but a surplus maximizing flow problem which we call the *single item, multi-unit procurement network formation (MPNF) problem*. This is the central problem explored in this paper.

A. Procurement Network Formation: How?

The literature on procurement network formation or supply chain formation can be classified into two broad areas - the first where we assume that agents in the network do not act strategically and the second, which is the focus of this paper, where they do act strategically to further their own payoffs. Broadly, the methodological approaches in addressing this problem has followed two tracks: the first uses techniques based on competitive equilibrium analysis [**?**], [**?**] and the second makes use of auction technology [**?**], [**?**], [**?**], [**?**], [**?**], [**?**], [**?**], [**?**], [**?**].

A recent article by Bajari, McMillan, and Tadelis [**?**] empirically compares the use of auctions versus negotiations in procurement and shows overwhelmingly that contractual relationships between agents in a procurement network are characterized by buyers and sellers often bargaining and negotiating over prices, quantities, delivery schedules, and several other attributes.

Bargaining and negotiation are well researched topics within the domain of cooperative game theory which, in several recent survey articles and handbook chapters [**?**], [**?**], [**?**] has been identified as an important tool in the supply chain researchers toolkit. Cooperative game theory has two central themes. The first theme is that of *feasible outcomes* and the second is that of *stability*. These two central themes of cooperative game theory have their perfect analogs in the context of procurement network formation. In procurement network formation, we are interested in constructing the feasible set of outcomes and picking the *best* from among them. In addition, as argued by management scientists, supply chains remain stable only when each of the agents gets a fair share of the surplus that the supply chain generates. So, we are interested in finding a fair way of dividing the gains that accrue when agents come together to create surplus value. In doing so, we are interested in finding a stable coalition of partners. Clearly then, cooperative game theory provides us with an apt toolkit to study the procurement network formation problem.

B. Research Contributions

In this paper, we focus on the multiple unit, single item procurement network formation problem when the agents who own edges in the procurement feasibility graph as well as the buyer are game theoretic agents. We assume that the agents are completely aware of (1) each others' costs and (2) the buyer's demand and valuation. Our specific contributions are as follows:

- We model the MPNF problem as a cooperative game with complete information, where the buyer is included as a game theoretic agent and characterize the consequence of using the Shapley value as a solution concept to the MPNF game.
- We then develop an extensive form game that implements the Shapley value of the MPNF game in sub-game perfect Nash equilibrium. Such an extensive form game can be embedded as a procurement network formation protocol.

II. The Model

As indicated earlier, the feasible network for forming the multiple unit, single item procurement network may be captured as a directed graph. We call this the procurement feasibility graph $G = (V, E)$, with V as the set of vertices's, two special nodes v_o (origin vertex) and v_t (terminal vertex), and $E \subseteq V \times V$ as the set of edges. With each of the edges $e \in E$ we associate the numbers $c(e)$, $l(e)$, and $u(e)$ to represent the cost, the lower bound, and the upper bound on the capacity of the edge respectively. Now, assume that each of the edges is owned by an agent i where i belongs to a finite set of agents $N = \{1, \ldots, n\}$. We define $\psi : E \rightarrow N$ such that $\psi(e) = i$ implies that agent i owns edge e. We let $\mathcal{I}(j)$ and $\mathcal{O}(j)$ represent the set of all incoming and outgoing edges at vertex $j \in V$. Note that we allow a single agent to own multiple edges.

Let $S \subseteq N$ be a coalition of agents. We let E_S represent the set of edges owned by agents in S. We also designate F_S as the flow in the network between the two special nodes v_o and v_t using only the edges E_S that are owned by agents in S. The flow on any edge $e \in E_S$ is designated as $f(e)$. For any flow F_S we denote the set of owners of the edges that facilitate the flow F_S as $\psi(F_S)$. We assume that if multiple units of the item are available to the buyer by using the flow F_S, then it costs $c(F_S)$ and the buyer is willing to compensate the edge owners with a value bF_S where b is the value that the buyer attaches to a single unit of the item. The surplus from such a transaction, is $bF_S - c(F_S)$. The maximum demanded quantity of the buyer is d_{v_t}. The problem now is to:

(a) Maximize the surplus $v_{\mu_b}(S)$ for $S = N$ which is done by solving the surplus maximizing optimization problem given in Equations (1) to (**??**):

$$v_{\mu_b}(S) \quad = \quad \max \; [bx_{v_t} - \sum_{e \in E_S} c(e)f(e)] \qquad (1)$$

subject to:

$$\sum_{e \in \mathcal{I}(j) \cap E_S} f(e) - \sum_{e \in \mathcal{O}(j) \cap E_S} f(e) = 0, \quad \forall j \in N \backslash \{v_o, v_t\} \tag{2}$$

$$\sum_{e \in \mathcal{I}(v_t) \cap E_S} f(e) = x_{v_t} \tag{3}$$

$$\sum_{e \in \mathcal{O}(v_o) \cap E_S} f(e) = x_{v_t} \tag{4}$$

$$0 \leq x_{v_t} \leq d_{v_t}, \quad \text{and} \quad l(e) \leq f(e) \leq u(e), \forall e \in E_S \tag{5}$$

(b) Divide the resulting surplus among the agents in the network in a fair way.

These two questions essentially constitute the multiple unit, single item procurement network formation problem. We denote this as $\mu_b = (G, N, \psi, b, d_{v_t})$ which in turn induces a cooperative game that can be represented in the characteristic function form as (N, v_{μ_b}) where N is the set of agents and v_{μ_b} is the characteristic function given by solving the optimization problem specified by Equations 1 to **??** for every $S \subseteq N$. We are now interested in finding solutions to this game.

III. The Shapley Value of the MPNF game: Key Issues

The Shapley value is an important single valued solution concept in cooperative game theory that has seen recent application to problems in supply chain management literature (see [**?**], [**?**], [**?**], [**?**], [**?**]). In this paper, we investigate the consequences of using the Shapley value to share the surplus that is generated when the procurement network is formed for multiple units of a single item from a given procurement feasibility graph.

A. The Shapley Value Allocations and Ownership Structure: An Example

To gain an understanding of how the Shapley value rule allocates surplus among the various agents in the MPNF game, let us first examine some illustrative procurement networks. We will later build on the intuition gained through these examples to formally show that the ownership structure plays an important role if the Shapley value is to make satisfactory allocations of the surplus in the context of the MPNF game.

Consider the procurement feasibility graphs T (top), L (left), and R (right) in Figure **??**. Here, we assume that each edge is owned by an independent agent. These agents are: 1, 2, and 3. The ownership is indicated by the differently hatched lines in the graphs. In addition we assume that the buyer (B) is also an agent with whom the surplus is to be shared.

For the procurement feasibility graphs T, L and R in Figure **??**, the edges are owned by agents 1 and 2, agents 1 and 2 and agents 1, 2 and 3 respectively. The data of the problems are shown on the graphs. From this data and from some simple calculations it is possible to generate the characteristic function fr each of the induced cooperative games. Folloiwng this the Shapley value allocations can be easily calculated and tey are as follows:

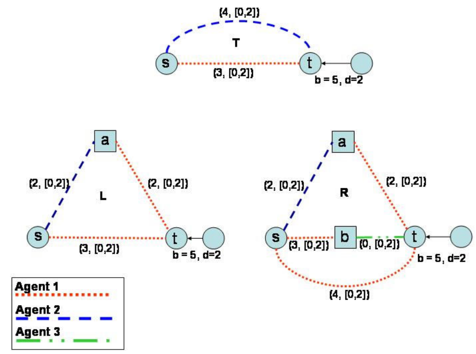

Fig. 3. Procurement graphs T, L, and R

The Shapley value allocations for agents 1,2, and B are for graph T are $4/3$, $2/6$ and $14/6$ respectively; for graph L the allocations for agents 1,2, and B are 2, 0 and 2 respectively; and for graph R the allocations for agents 1,2, 3, and B are $9/6$, $1/6$, $5/6$ and $9/6$ respectively.

There is an interesting observation to make from the examples above. Observe that in the procurement feasibility graph T, the surplus maximizing transaction can be carried out by agents 1 and B without the help of agent 2. Similarly, in the procurement feasibility graph R, the surplus maximizing transaction can be carried out by agents 1, 3, and B without the help of agent 2. Yet, notice that in the Shapley value allocations for these graphs, non-zero allocations are being made to agents who are not part of the set that generates a *surplus maximizing* transaction. This can be disconcerting in a procurement situation where the surplus allocation rule gives away money to agents who do not actually participate in the surplus maximizing flow. It is therefore important for us to gain a characterization of the scenarios where the Shapley Value makes allocations of surplus value to the buyer and only those agents who own edges in the surplus maximizing flow.

IV. A Result on Shapley Value Allocations and the Ownership Structure

The thrust of the analysis in this section is in providing the above characterization. Before that, we need the following definition and proposition to ease the discussion that follows.

Definition 1: The set of all agents, denoted $S_M(F)$, who own edges in a surplus maximizing flow F of the procurement graph are called the SM-Agents, i.e., $S_M = \{i = \psi(e), i \in N : e \in \psi(F)\}$ associated with the surplus maximizing flow F.

In the sequel, for ease of discussion we assume that there is a unique surplus maximizing flow. For multiple surplus maximizing flows, the analysis can be easily extended.

The MPNF game has a specialized property called a zero monotonic game which we capture as a proposition below. To economize on space, we have chosen to omit the proofs of all the propositions and theorems in this paper. We refer the reader [**?**] for a detailed exposition of the omitted parts.

Proposition 1: The characteristic function of the MPNF game when the buyer is included as an agent is zero-monotonic, i.e., $v_{\mu_b}(N) \geq v_{\mu_b}(N \setminus \{i\}) + v_{\mu_b}(\{i\}), \forall i \in N$. We now state and prove the main theorem of this section.

Theorem 1: If the Shapley value rule allocates all the surplus value in the MPNF game only to agents $i \in S_M$ then for every flow F_S provided by a coalition S that includes an agent $i \notin S_M$, either

1) F_S is not profitable, i.e, $v_{\mu_b}(S) = 0$ or
2) if F_S is profitable, then we have $\psi(F_S) \cap S_M \neq \emptyset$, and there is a set $S_{S_M} \subset \psi(F_S) \cap S_M$ such that $v(S_{S_M}) = v(\psi(F_S))$.

Proof: Assume that the Shapley value allocates all the surplus value only to agents in the set S_M. i.e., the surplus $x = (\widehat{x}_i)_{i \in N}$ is such that $\sum_{i \in S_M} x_i = v_{\mu_b}(N)$ and $x_i = 0, \forall i \in \widehat{S}_M$ where $\widehat{S}_M = N \setminus S_M$.

The Shapley value rule for the MPNF game is given by

$$x_i = \sum_{S \subseteq N,\, S \ni i} \frac{(|S|-1)!(|N|-|S|)!}{|N|!} \left[v_{\mu_b}(S) - v_{\mu_b}(S \setminus \{i\})\right] \quad (6)$$

Since the Shapley value allocates all of the surplus without wasting any of the resource, we can write the following:

$$\sum_{i \in N} x_i = \sum_{i \in N} \sum_{S \subseteq N,\, S \ni i} \frac{(|S|-1)! \times (|N|-|S|)!}{|N|!} \left[v_{\mu_b}(S) - v_{\mu_b}(S \setminus \{i\})\right] = v_{\mu_b}(N) \quad (7)$$

This can be written as

$$\sum_{i \in S_M} \sum_{S \subseteq N,\, S \ni i} \frac{(|S|-1)! \times (|N|-|S|)!}{n!} \left[v_{\mu_b}(S) - v_{\mu_b}(S \setminus \{i\})\right] + \sum_{i \in \widehat{S}_M} \sum_{S \subseteq N,\, S \ni i} \frac{(|S|-1)! \times (|N|-|S|)!}{|N|!} \left[v_{\mu_b}(S) - v_{\mu_b}(S \setminus \{i\})\right] = v_{\mu_b}(N) \quad (8)$$

From our assumption that the Shapley value allocates all the surplus to only agents in the set S_M, the first term may be replaced as follows:

$$v_{\mu_b}(N) + \sum_{i \in \widehat{S}_M} \sum_{S \subseteq N,\, S \ni i} \frac{(|S|-1)! \times (|N|-|S|)!}{|N|!} \left[v_{\mu_b}(S) - v_{\mu_b}(S \setminus \{i\})\right] = v_{\mu_b}(N) \quad (9)$$

$$\Rightarrow \sum_{i \in \widehat{S}_M} \sum_{S \subseteq N,\, S \ni i} \frac{(|S|-1)! \times (|N|-|S|)!}{|N|!} \left[v_{\mu_b}(S) - v_{\mu_b}(S \setminus \{i\})\right] = 0 \quad (10)$$

Notice that since $v_{\mu_b}(S)$ is a monotonically non-decreasing function, every term in equation (**??**) must be zero. i.e.,

$$\frac{(|S|-1)! \times (|N|-|S|)!}{|N|!} \left[v_{\mu_b}(S) - v_{\mu_b}(S \setminus \{i\})\right] = 0, \forall i \in \widehat{S}_M, \forall S \subseteq N, S \ni i \quad (11)$$

$$\Rightarrow \left[v_{\mu_b}(S) - v_{\mu_b}(S \setminus \{i\})\right] = 0, \forall i \in \widehat{S}_M, \forall S \subseteq N, S \ni i \quad (12)$$

In equation (**??**), we need to analyse two cases: For each $i \in \widehat{S}_M, \forall S \subseteq N, S \ni i$, either $v_{\mu_b}(S) = 0$ or $v_{\mu_b}(S) > 0$.

Case 1: When $v_{\mu_b}(S) = 0$. Because $v_{\mu_b}(S)$ is a monotonically non-decreasing function, we have $v_{\mu_b}(S \setminus \{i\}) = 0$. Also, since $v_{\mu_b}(S) = 0$, we can infer that the coalition S does not own edges which can provide a profitable flow in the network. So, condition 1 of the theorem holds in this case.

Case 2: When $v_{\mu_b}(S) > 0$. Because the term $[v_{\mu_b}(S) - v_{\mu_b}(S \setminus \{i\})] = 0$, we must have $v_{\mu_b}(S) = v_{\mu_b}(S \setminus \{i\})$. Now the set $S \setminus \{i\}$ may be of three types: (a) it may contain only agents from the set S_M, (b) it may contain agents only from the set $\widehat{S}_M$ or (c) it may contain agents from both sets S_M and $\widehat{S}_M$.

Case (2.a): The theorem holds in this case because we have found a coalition of agents belonging to S_M who provide the same surplus as the surplus provided by coalition S.

Case (2.b): This is impossible because of our construction of the network. The buying agent always owns an edge that is common to all (v_o, v_d) flows in the network and this agent is always in the set S_M.

Case (2.c): Let $\bar{S} = S \setminus i$. We know from equation (**??**) that $v_{\mu_b}(S) = v_{\mu_b}(\bar{S}) > 0$. We can now pick an agent j from $\bar{S}$ such that $j \in \widehat{S}_M$. Now, for any such $j \in \bar{S}; j \in \widehat{S}_M$ and $\bar{S}$, we know from equation **??** that $v_{\mu_b}(\bar{S}) - v_{\mu_b}(\bar{S} \setminus j) = 0$. So, we have $v_{\mu_b}(\bar{S}) = v_{\mu_b}(\bar{S} \setminus j) = v_{\mu_b}(S) > 0$. Since there are a finite number of agents, we can carry out this step a finite number of times before one of two things happen - either $\bar{S} \subseteq S_M$ or $\bar{S} = \emptyset$. If it is the former, then we are done. If it is the latter, we know that $v_{\mu_b}(\emptyset) = 0$ and hence we must have started out with a set $\bar{S}$ which generated a surplus value of 0 which is a contradiction. So $\bar{S}$ must have been a subset of S_M. So, condition 2 of the theorem is satisfied in this case. This proves the theorem. ■

V. An Extensive Form Game to Implement the Shapley Value

We now present an extensive form game that is inspired by the mechanisms in [**?**], [**?**], to implement the Shapley value of the MPNF game.

A. Description of the Extensive Form Game

The extensive form game that we construct to realize the Shapley value as an equilibrium outcome is essentially a three stage game. In Stage 1, the agents move simultaneously and in Stages 2 and 3, they move sequentially. The building blocks of the extensive form game to implement the Shapley value of the MPNF game are as follows:

1) A finite set of agents N, a finite set of nodes $\mathcal{N}$, and a finite set of actions $\mathcal{A}$.
2) The set of agents $N = \{0, 1, 2, \ldots, n\}$ corresponds to the buying agent 0 and the agents $\{1, 2, \ldots, n\}$ who own edges in the MPNF scenario $\mu_b = (G', N, \psi, b, d_{v_t})$.
3) A function $pr : \mathcal{N} \rightarrow (\mathcal{N} \cup \emptyset)$ specifying a single immediate predecessor $pr(k)$ of each node $k \in \mathcal{N}$. It is nonempty for all $k \in \mathcal{N}$ except for one node which

is designated as the initial node k^0. The immediate successor nodes of k are then $su(k) = pr^{-1}(k)$ and the set of all predecessors and successors can be found by iterating $pr(n)$ and $su(n)$. To have a tree structure, we require that these sets be disjoint, i.e. a predecessor of a node cannot also be a successor of the node. The set of leaf nodes of the tree is $\mathcal{L} = \{k \in \mathcal{N} | su(k) = \emptyset\}$. All other nodes $\mathcal{N} \setminus \mathcal{L}$ are called decision nodes. These decision nodes may be divided into three sets $\mathcal{N}^1$, $\mathcal{N}^2$, and $\mathcal{N}^3$ which contain the decision nodes in stages 1, 2, and 3 of the game respectively.

4) The action set $\mathcal{A}$ consists of three sets: $\mathcal{A}^1$ for Stage 1; $\mathcal{A}^2$ for Stage 2 and $\mathcal{A}^3$ for Stage 3. So, $\mathcal{A} = \mathcal{A}^1 \cup \mathcal{A}^2 \cup \mathcal{A}^3$.
   a) In Stage 1, each of the players from $N = \{0, 1, 2, \ldots, n\}$ chooses a bid vector $b_i = (b_{ij})_{j \in N \setminus \{i\}}$ where $b_{ij} \in \Re$. The bid b_{ij} represents the value that agent i is willing to transfer to agent j in order to become the proposer. So, the elements of the action set $\mathcal{A}^1$ are as follows:
   $$\mathcal{A}^1 = \{b_i | b_i \in \Re^{|N|}; b_{ij} \in \Re\}.$$
   Now, for each $i \in N$, we define the net bid B_i as $B_i = \sum_{j \in N \setminus \{i\}} b_{ij} - \sum_{j \in N \setminus \{i\}} b_{ji}$. The net bid of an agent gives us a measure of the relative importance that each of the agents attaches to becoming the proposer. We now designate β as the agent who has the highest net bid, i.e., β = argmax $_{i \in N}(B_i)$. In case of a tie, the agent β is chosen randomly from among the agents who have the highest net bid. Once the agent β is chosen he pays the bid amounts as specified in his bid vector $b_{\beta j}$ to each of the agents $j \neq \beta$.
   b) In Stage 2, agent β makes an offer (N, ω_β) to coalition N to implement an outcome $\omega_\beta \in \Omega_N$. The outcome ω_β is a tuple $(r_{\beta j}, x_{\beta j})_{j \in N}$. $(r_{\beta j})_{j \in N}$ indicates a reassignment of the edges and capacities so that the procurement network is formed and $(x_{\beta j})_{j \in N}$ is a division of the surplus among agents in N that agent β is willing to concede. So, the elements of the action set $\mathcal{A}^2$ are as follows:
   $$\mathcal{A}^2 = \{\text{make offer}(N, \omega_\beta) | \omega_\beta \in \Omega_N; \omega_\beta = (r_{\beta j}, x_{\beta j})_{j \in N}\}.$$
   c) In Stage 3, all players except β, either accept or reject the Stage 2 proposal sequentially. So, the elements of the action set $\mathcal{A}^1$ are as follows:
   $$\mathcal{A}^3 = \{\text{accept offer}(N, \omega_\beta), \text{reject offer}(N, \omega_\beta)\}$$
   If the offer is accepted by all the agents $i \in N \setminus \{\beta\}$, then each of these agents transfers the edges and capacities according to the proposal $(r_{\beta j})_{j \in N}$ and in return receives a share of the surplus $(x_{\beta j})_{j \in N}$ as indicated in the offer ω_β.
   If the offer is rejected, then all agents other than β replay the game. So, now the set of agents participating in the game are $N \setminus \{\beta\}$ and agent β simply gets his stand alone allocation, which is 0.

5) A function $\alpha : \mathcal{N} \setminus \{k_0\} \rightarrow \mathcal{A}$ gives the action that leads to any non-initial node k from its immediate predecessor node $pr(k)$ and satisfies the property that if $\widehat{k}, \bar{k} \in su(k)$ and $\widehat{k} \neq \bar{k}$, then $\alpha(\widehat{k}) \neq \alpha(\bar{k})$. The set of actions available at decision node k is $a(k) = \{a \in \mathcal{A} : a = \alpha(\bar{k}), \text{ for some } \bar{k} \in su(k)\}$.
6) A collection of information sets $\mathcal{H}$, and a function $H : \mathcal{N} \rightarrow \mathcal{H}$ assigning each decision node to an information set $H(n) \in \mathcal{H}$. Thus the information sets $\mathcal{H}$ make a partition of the set of nodes $\mathcal{N}$. We require that all decision nodes assigned to a single information set have the same actions available at every one of those nodes. That is, $a(k) = a(\widehat{k})$ if $H(k) = H(\widehat{k})$. We can therefore write the actions available at information set H as $A(H) = \{a \in \mathcal{A} : a \in a(k) \text{ for } k \in H\}$.
7) A function $\mathcal{I} : \mathcal{H} \rightarrow N$, assigning each information set in $\mathcal{H}$ to the agent who moves at the decision nodes in that set. We can denote the collection of agent i's information sets by $\mathcal{H}_i = \{H \in \mathcal{H} : i = \mathcal{I}(\mathcal{H})\}$.
8) A strategy for an agent i is a function $s_i : \mathcal{H}_i \rightarrow \mathcal{A}$ such that $s_i(H) \in A(H)$ for all $H \in \mathcal{H}_i$. We now denote
   - the set of all possible strategies for an agent i by $\mathcal{S}_i$.
   - the strategy profile for the coalition of all agents in N as $s = (s_0, s_1, s_2, \ldots, s_n)$.
   - the set of all possible strategy profiles of all agents in N as $\mathcal{S} = \times_{i \in N} \mathcal{S}_i$
9) A function $g : \mathcal{S} \times \mathcal{N} \rightarrow \Omega$ where $\mathcal{S}$, $\mathcal{N}$, and Ω are the set of strategy profiles, set of nodes in the game tree, and the set of outcomes respectively. Note that for each of the leaf nodes $\mathcal{L}$ corresponds to an outcome from the set Ω. So, for a strategy profile $s \in \mathcal{S}$, we let $g(s, k)$ denote the outcome corresponding to s starting at node k.
10) Finally we define a collection of payoff functions $u = \{u_0(\cdot), u_1(\cdot), u_2(\cdot), \ldots, u_n(\cdot)\}$ that agents have for each of the outcomes. That is $u_i : \Omega \rightarrow \Re$ and $u : \Omega \rightarrow \Re^{|N|}$. We know that an outcome $\omega \in \Omega$ is given by the tuple $\omega = (r_i, x_i)_{i \in N}$. We let the utility that agent i has for an outcome $\omega \in \Omega$ be $u_i(\omega)$. We define the utilities as follows:
    If the offer ω made by agent β is accepted by all other agents then:
    - $u_\beta(\omega) = v_{\mu_b}(N) - \sum_{j \neq \beta}[b_{\beta j} + x_{\beta j}]$, i.e., the agent β obtains the value of the coalition N minus the sum of the payments and the bids made in Stage 1.
    - $u_i(\omega) = b_{\beta j} + x_{\beta j}$, i.e., the net payment that an agent $j \neq \beta$ receives is the sum of the bid values in Stage 1 and the transfers proposed in Stage 2 by agent β.

    If the offer ω made by agent β is rejected by any one of the remaining agents then:
    - $u_\beta(\omega) = v_{\mu_b}(\{\beta\}) - \sum_{j \neq \beta} b_{\beta j}$, i.e, the agent β

obtains his standalone value minus the bids that he made in Stage 1.

- $u_i(\omega) = b_{\beta j} + \Phi_i((N \setminus \{\beta\}), v_{\mu_b})$ where $\Phi_i((N \setminus \{\beta\}), v_{\mu_b})$ is the value that the agent gets from the game $(N \setminus \{\beta\}, v_{\mu_b})$ played without agent β and $b_{\beta j}$ is the initial bid mount from agent β.

So, given the MPNF game in characteristic function form (N, v_{μ_b}), the extensive form game Γ is specified by the tuple:

$$\Gamma = (\mathcal{N}, \mathcal{A}, N, pr(\cdot), \alpha(\cdot), \mathcal{H}, H(\cdot), \mathcal{I}(\cdot), u) \tag{13}$$

B. Analysis of the Extensive Form Game

The analysis of the mechanism essentially involves verifying if the extensive form game Γ constructed above makes the agents in the game pick equilibrium strategies whose payoffs correspond to the Shapley value allocations of the corresponding game specified in characteristic function form. We do this by proving Theorem **??** below.

The theorem essentially says that in the extensive form game, the agents' payoffs for any sub-game perfect Nash equilibrium strategy profile is the same as the Shapley value allocations in the MPNF game specified in the characteristic function form. Simultaneously, it also says that the Shapley value allocations of the MPNF game specified in characteristic function form can always be obtained through an equilibrium of the extensive form game. We now state the theorem without a proof which is available in [**?**].

Theorem 2: The extensive form game $\Gamma = (\mathcal{N}, \mathcal{A}, N, pr(\cdot), \alpha(\cdot), \mathcal{H}, H(\cdot), \mathcal{I}(\cdot), u)$ implements the Shapley value of the MPNF game in sub-game perfect Nash equilibrium (SPNE).

Proof: For reasons of space we omit the proof of this theorem and refer the interested reader to [**?**] for a detailed exposition.

VI. Discussion and Conclusion

In the context of the procurement problem where we are trying to form the procurement network from a given set of feasible options (feasibility graph), we would like to ensure that the surplus is divided only among the agents who provide the resources to maximize the surplus. Theorem **??** provides us with a characterization of the network structure which tells us that if the Shapley value rule were to be used to divide the surplus so that the objective indicated above is realized then a certain set of agents need to own a critical set of edges in the procurement feasibility graph. In addition we require that this subset of agents are also capable of providing a flow which generates the same surplus.

The analysis through Theorem **??** shows us that in addition to realizing the Shapley value, the mechanism provides us with an explicit form of the equilibrium strategies. That is, in the bidding stage, each agent agrees to transfer to every other agent a value commensurate with the difference between the Shapley values that he would get in the original game and with the agent removed. Then in Stage 2, the agent who is chosen to be the proposer simply makes an offer which gives to every other agent the balance of the Shapley value allocation that was withheld in the first stage. The easy way in which these strategies can be computed gives us greater confidence in believing that the mechanism is a credible way of forming procurement networks in a distributed manner. Another important thing to notice is that the agreements can be reached with a finite number of message exchanges because the extensive form game itself is finite.

References

[1] W. Walsh and M. Wellman, "Decentralized supply chain formation: A market protocol and competitive equilibrium analysis," *Journal of Artificial Intelligence*, vol. 19, pp. 513–567, 2003.

[2] ——, "Efficiency and equilibrium in task allocation economies with hierarchical dependencies," in *Sixteenth International Joint Conference on Artificial Intelligence*, 1999, pp. 520–526.

[3] M. Babaioff and N. Nisan, "Concurrent auctions across the supply chain," *Journal of Artificial Intelligence Research*, vol. 21, pp. 595–629, May 2004.

[4] D. Garg, "Design of innovative mechanisms for contemporary game theoretic problems in eletronic commerce," Ph.D. dissertation, Department of Computer Science and Automation, Indian Institute of Science, June 2006.

[5] W. Walsh, M. Wellman, and F.Ygge, "Combinatorial auctions for supply chain formation," in *Proceedings of ACM Conference on Electronic Commerce (EC-00)*, 2000, pp. 260–269.

[6] W. Walsh, "Market protocols for decentralized supply chain formation," Ph.D. dissertation, Department of Computer Science and Engineering, University of Michigan, Ann Arbor, 2001.

[7] M. Babaioff and W. Walsh, "Incentive-compatible, budget-balanced, yet highly efficient auctions for supply chain formation," *Decision Support Systems*, vol. 39, pp. 123–149, March 2005.

[8] S. Jiong and N. Sadeh, "Coordinating multi-attribute procurement auctions subject to finite capacity considerations," School of Computer Science, Carnegie Mellon Unviversity, CMU-ISRI-03-105 Tech. Report, Tech. Rep., 2004.

[9] J. Collins and M. Gini, "A testbed for multi-agent contracting for supply chain formation," in *IJCAI-2001 Workshop on Artificial Intelligence and Manufacturing*, 2001.

[10] J. Collins, "Solving combinatorial auctions with temporal constraints in economic agents," Ph.D. dissertation, Department of Computer Science and Engineering, University of Minnesota, 2002.

[11] M. Fan, J. Stallert, and A. Whinston, "Decentralized mechanism design for supply chain organizations using an auction market," *Information Systems Research*, vol. 14, no. 1, pp. 1–22, March 2003.

[12] P. Bajari, R. McMillan, and S. Tadelis, "Auctions versus negotiations in procurement: An empirical analysis," NBER Working Paper Series, Department of Economics, Stanford University, Tech. Rep., 2003.

[13] G. P. Cachon and S. Netessine, *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*. Kluwer Academic Publishers, 2004, ch. Game Theory in Supply Chain Analysis.

[14] M. Nagarajan and G. Sosic, "Game-theoretic analysis of cooperation among supply chain agents: Review and extensions," *European Journal of Operations Research (accepted for publication in 2006)*, 2006.

[15] J. Bartholdi and E. Kemahhoglu-Ziya, "Using Shapley value to allocate savings in a supply chain," in *Supply Chain Optimization, Kluwer Optimization Series, Editors: P. Pardolos and J. Geunes*, 2004.

[16] E. Kemahhoglu-Ziya, "Formal methods of value sharing in supply chains," Ph.D. dissertation, School of Industrial and Systems Engineering, Georgia Institute of Technology, July 2004.

[17] D. Granot and G. Sosic, "A three stage model for a decentralized distribution system of retailers," *Operations Research*, vol. 51, no. 5, pp. 771–784, 2003.

[18] ——, "Formation of alliances in internet-based supply exchanges," *Management Science*, vol. 51, no. 1, pp. 92–105, 2005.

[19] T. Chandrashekar and Y. Narahari, "Procurement network formation: A cooperative game theoretic approach," Indian Institute of Science, Tech. Rep., 2006.

[20] V. Krishna and R. Serrano, "Perfect equlibria of a model of n-person non-cooperative bargaining," *International Journal of Game Theory*, vol. 24, pp. 259–272, 1995.

[21] D. Perez-Castrillo and D. Wettstein, "Bidding for the surplus: A non-cooperative approach to the Shapley value," *Journal of Economic Theory*, vol. 100, pp. 274–294, 2001.