A New Approach to the Design of Electronic Exchanges

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Abstract. Electronic Exchanges are double-sided marketplaces that allows multiple buyers to trade with multiple sellers, with aggregation of demand and supply across the bids to maximize the revenue in the market. In this paper, we propose a new design approach for an one-shot exchange that collects bids from buyers and sellers and clears the market at the end of the bidding period. The main principle of the approach is to decouple the allocation from pricing. It is well known that it is impossible for an exchange with voluntary participation to be efficient and budget-balanced. Budget-balance is a mandatory requirement for an exchange to operate in profit. Our approach is to allocate the trade to maximize the reported values of the agents. The pricing is posed as payoff determination problem that distributes the total payoff fairly to all agents with budget-balance imposed as a constraint. We devise an arbitration scheme by axiomatic approach to solve the payoff determination problem using the added-value concept of game theory.

1 Introduction

Markets play a central role in any economy and facilitate the exchange of information, goods, services, and payments. They are intended to create value for buyers, sellers, and for society at large. Markets have three main functions [1]:matching buyers to sellers; facilitating exchange of information, goods, services, and payments associated with a market transaction; and providing an institutional infrastructure. Internet-based E-markets leverage information technology to perform these functions with increased effectiveness and reduced transaction costs, leading to more efficient, friction-free markets. In this paper, our interest is in providing a new approach to the design of exchanges. Exchanges allows multiple buyers to trade with multiple sellers, with aggregation of demand and supply across the bids to maximize the revenue in the market. The approach is to decouple allocation from pricing. This is not a new approach as Vickrey

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mechanisms essentially do the same. However, our objectives in the design are budget-balance and individual rationality rather than efficiency and incentive compatibility. Our approach is in the line with the constructive approach to mechanism design in [11], but our pricing scheme is different. We propose an arbitration scheme for pricing, based on an axiomatic approach that captures the intuitive relationships between the payoff and added-value concept of game theory.

2 Electronic Exchanges

Exchanges are double-sided marketplaces where both buyers and sellers submit bids for trading. We refer sellers' bids as *offers* if required to differentiate them from that of buyers. The exchanges differ in functionality with respect to timing of clearing, number of bid submissions, pricing, aggregation and the varieties of goods traded. In this paper, our interest is in designing one-shot exchanges, where buyers and sellers submit bids during the bidding period and the market is cleared at the end of the bidding interval. This is similar to *call markets* or *clearing house* [6] but the bids may have multiple attributes and can be combinatorial in nature.

2.1 Design Issues

The two core problems in an exchange are *allocation* and *pricing* [11]. In certain exchanges pricing determines the allocation. In call markets [6], the price at which there is a maximal match in supply and demand is used to clear the market. But there are exchanges that decouple allocation from pricing. For e.g., the Generalized Vickrey Auction (GVA) [13] allocates the goods that maximizes the reported bid values and pricing is done based on allocation. Our approach is to decouple allocation from pricing but with a different set of objectives than that of GVA. We call the allocation problem as the *trade determination problem* (TDP) and the pricing problem as the *payoff determination problem.* TDP determines the goods traded between every pair of buyers and sellers and the PDP prices the goods traded by determining the *payoff* (to be defined later) to individual agents.

3 Impossibility Result

Useful economic properties of an exchange include [11]:

- **Allocative-efficiency** (EFF): Trade should be executed to maximize the total increase in value over all agents.
- **Individual-rationality** (IR): No agent should pay more than its net increase in value for the items it trades.
- **Budget-balance** (BB): The total payments received by the exchange from agents should be at least the total payments made by the exchange to agents.

– **Incentive-compatibility** (IC): The equilibrium strategy for the agents is to reveal *truthful* information about their preferences in their bids.

The impossibility result of Myerson & Satterthwaite [10] demonstrates that no exchange can be EFF, BB and IR. This result holds with or without IC and for both dominant strategy and Bayesian-Nash equilibrium [11]. For impossibility results regarding the incompatibility of dominant strategies, EFF and BB refer [4][5].

3.1 Efficiency and Incentive Compatibility

Allocative-efficiency ensures that the allocation is Pareto-efficient and there are no resale of goods among the agents. In general, IC is not necessary for EFF, e.g. open-cry English auction. But for one-shot exchanges, IC is both necessary and sufficient for EFF. This can be easily seen as the one-shot exchanges obtain bids from agents only once and there is no other way to know the agents' preferences. The use of exchanges in electronic commerce for procurement and as intermediaries requires BB and IR as mandatory properties [11]. Without BB, the market-maker cannot operate the exchange in profit and without IR, agents cannot participate voluntarily. By the impossibility result we have to allocate goods inefficiently with BB and IR. So in our approach we relax EFF. The constructive approach to mechanism design developed in [11] adopt a similar approach, however the pricing problem is modeled as a mathematical programming problem to determine the pricing function that is in minimum *distance* from the VGA pricing and they investigate the use of different distance norms. We do not impose IC as a hard constraint for the following reasons:

– For the one-shot exchange, IC is desirable to implement EFF, but with BB and IR as hard constraints, EFF is not possible even with IC.
– IC is also important from the agents' perspective, as it avoids game-theoretic deliberation about other agents' strategies [13]. However, IC is useful only for private valued goods, as agents will not declare their true valuation to avoid winner's curse for common valued goods [12]. Use of exchanges in electronic commerce are mainly for procurement and brokerage which deal with correlated and common valued goods and hence IC will not help the agents much in the exchanges. It is to be noted here that GVA, in addition to being IC, avoids winner's curse. But unfortunately, GVA is not BB [11].
– IC very much depends on the allocation algorithm. The allocation problem for many exchanges are in general NP-hard and the IC that holds for optimal allocations generally breaks for approximate solutions [7]. We need to develop different IC mechanisms for different approximation schemes.
– The *revelation principle* [9] asserts that whatever that can be achieved by a non-incentive-compatible mechanism can be achieved by a IC direct mechanism. So we can indeed create a IC direct mechanism. However, it is not feasible in practice as the revelation principle makes unreasonable assumptions about the computational capabilities of the agents [11].

Thus we are interested in designing a BB and IR one-shot electronic exchanges. In the following sections we progressively describe the TDP and PDP.

4 Trade Determination Problem

TDP determines the goods traded between every buyer-seller pair such that the revenue in the market is maximized, by an allocation that maximizes the reported values (in the bids and offers) of the agents. This is desirable to avoid ex post claims from participants that an efficient trade was forfeited [11]. Such a revenue maximizing allocation need not be EFF and it is impossible to verify EFF if we do not have truth revelation from the agents. We model the TDP as a mathematical programming problem.

4.1 Notation

The following notation will be used for the rest of the paper. The notation is general enough to include combinatorial bids. Without loss of generality we will assume that an agent submits only one bid.

$\mathcal{B}$ Set of bids, $\{1, 2, \ldots, m, \ldots, M\}$
$\mathcal{S}$ Set of offers, $\{1, 2, \ldots, n, \ldots, N\}$
$\mathcal{G}$ Set of Goods $K \subseteq \mathcal{G}$
$WTP_m(K) \in \mathbb{R}_+ \setminus \{0\}$, *Willingness-to-Pay*: Maximum unit price a buyer m is willing to pay for K
$WTS_n(K) \in \mathbb{R}_+ \setminus \{0\}$, *Willingness-to-Sell*: Minimum unit price a seller n is willing to accept for K
$y_{mn}(K) \in \mathbb{Z}_+$ Number of goods of K traded between agents m and n

TDP is the following mathematical programming problem:

$$R = \max \sum_{Feasible(m,n,K)} (WTP_m(K) - WTS_n(K))y_{mn}(K) \tag{1}$$

$$\text{subject to: } Feasible(y_{mn}(K))$$

where $Feasible(m, n, K)$ is the set of compatible bids with respect to the defined attributes and $WTP_m(K) \geq WTS_n(K)$. $Feasible(y_{mn}(K))$ is the set of feasible trades with respect to supply-demand constraints and aggregation constraints. TDP determines the optimal trade $y^*_{mn}(K)$ that maximizes the reported surplus and the maximum possible surplus out of all allocations is given by R in (1). This problem can be NP-hard depending on the set $Feasible(y_{mn}(K))$.

5 Payoff Determination Problem

The pricing problem in an exchange is to determine the payments made by the agents to the exchange and vice-versa after the exchange clears.

Definition 1 (Trade). *Trade of a buyer* m *is the* $|\mathcal{G}|$ *tuple* $\alpha_m = (\alpha_m^1, \ldots, \alpha_m^{|\mathcal{G}|})$, *where* $\alpha_m^i = \sum_{K:i \in K} \sum_n y_{mn}(K)$. *Similarly, trade of a seller* n *is the* $|\mathcal{G}|$ *tuple* $\beta_n = (\beta_n^1, \ldots, \beta_n^{|\mathcal{G}|})$, *where* $\beta_n^i = \sum_{K:i \in K} \sum_m y_{mn}(K)$

Definition 2 (Payment). *The payment* $p_m(\alpha_m)$ *to buyer* m *is the price paid by the buyer to the exchange if* $p_m(\alpha_m) > 0$ *and vice-versa if otherwise. The payment* $q_n(\beta_n)$ *to seller* n *is the price paid by the exchange to the seller if* $q_n(\beta_n) > 0$ *and vice-versa if otherwise.*

Definition 3 (Value of Trade). *The value* $v_m(\alpha_m)$ *is the monetary value of the trade* α_m *to buyer* m *and value* $u_n(\beta_n)$ *is the monetary value of the trade* β_n *to the seller* n.

The values of the trade are private information to the agents and will not be revealed unless the mechanism is IC. Agents will instead strategically reveal $\tilde{v}_m(\alpha_m)$ and $\tilde{u}_n(\beta_n)$ to the exchange in their bids and offers, respectively. With the revealed values, we implement IR as following constraints:

$$\tilde{v}_m(\alpha_m^*) - p_m(\alpha_m^*) \geq 0, \; \forall \text{ bids } m \tag{2}$$

$$q_n(\beta_n^*) - \tilde{u}_n(\beta_n^*) \geq 0, \; \forall \text{ offers } n \tag{3}$$

where α_m^* and β_n^* are allocated trades when the exchange clears. The above equations are satisfied at equality when the agents pay their quoted price. For BB, we require:

$$\sum_m p_m(\alpha_m^*) = \sum_n q_n(\beta_n^*) \tag{4}$$

The exchange operates at no profit with the above BB, but it can be assumed without loss of generality as the exchange can extract a fixed percentage of the payment to the agents as revenue.

Definition 4 (Payoff). *The payoff* Δ_m *to buyer* m *is* $\tilde{v}_m(\alpha_m^*) - p_m(\alpha_m^*)$ *and the payoff* Θ_n *to seller* n *is* $q_n(\beta_n^*) - \tilde{u}_n(\beta_n^*)$.

The payoff is the real utility to the agents *only* when the revealed values are the true values. With respect to payoffs, IR can be written as:

$$\Delta_m \geq 0 \; \forall \text{ bids } m \tag{5}$$

$$\Theta_n \geq 0, \; \forall \text{ offers } n \tag{6}$$

and BB as:

$$\sum_m \tilde{v}_m(\alpha_m^*) - \sum_n \tilde{u}_n(\beta_n^*) = \sum_m \Delta_m + \sum_n \Theta_n \tag{7}$$

It can be easily seen that $\sum_m \tilde{v}_m(\alpha_m^*) - \sum_n \tilde{u}_n(\beta_n^*) = R$, which is the maximized reported surplus given by TDP. The individual payoffs are distributed from this surplus. The exchanges are double-sided markets as both buyers and sellers submit bids. It is reasonable to assume that the exchange should be neutral to both the buyers and sellers in distributing the payoffs, so that it is attractive to both of them.

Definition 5 (Neutrality). *The exchange is neutral if* $\sum_m \Delta_m = \sum_n \Theta_n$.

There are infinite possible values for Δ_m and Θ_n that satisfy (7) and neutrality. For e.g. the mid-point pricing scheme that prices a trade between a buyer with bid value WTP_m, and a seller with offer value WTS_n, at the mid-point $(\frac{WTP_m+WTS_n}{2})$ satisfies the above conditions. The general approach is to determine the payoff through pricing, but we take the reverse approach i.e. price the agents based on the payoff.

Definition 6 (PDP). *The payoff determination problem (PDP) is to determine individual payoffs* Δ_m *to buyers and* Θ_n *to sellers such that (7) and neutrality are satisfied.*

The payoff determination and pricing are equivalent in the sense that one can be derived from the other. However, payoff determination approach handles certain critical issues that cannot be handled by pricing. In all market mechanisms different pricing schemes were only used to investigate IC and existence of dominant strategy equilibriums. These are sufficient if a single seller trades with a single buyer, like traditional auctions. Most of the research is focussed on how to carry over these properties of single-sided auctions to electronic exchanges with multiple buyers and multiple sellers (like GVA from Vickrey auctions). But the overlooked point is that electronic exchanges possibly allow trading multiple units of different goods based on multiple attributes with aggregation over supply and demand. A pricing scheme that achieves IC or dominant strategy equilibrium may not consider the *value* added by a bid in the current trade and hence may not *fairly* distribute the payoff to the bids.

Consider a multi-attribute procurement exchange with attributes like price, quantity, delivery time, post-sale service, warranty etc. Let there be a single seller who wants to procure some products with some desirable value for the above attributes. Bids are accepted from the buyers and let five bids be finally selected for trading that are compatible in all the attributes and also maximize the profit for the seller. Let us assume that we somehow calculated the total payoff to all the buyers as 100. How can we distribute this total payoff to individual buyers? One naive approach is to give a non-negative weight and a value to each attribute (where the weights sum up to unity) and to obtain a single numerical quantity that quantifies a bid, by the weighted average method. Then using these numerical quantities we can find the relative importance of each bid. However, it is not trivial to determine a weight and value to each attribute (some attributes like warranty, logistics are not easily quantifiable) that can convince all the agents as being *fair*. We will present an alternate approach to determine the relative importance of each bid using the concept of *added value*.

5.1 Added Value

The *added value* of a player in the game is the difference of the values of the game when the player is *in* in the game and when he is *out* of the game [2][3]. Added value measures what each player brings to the game. Added value concept is extensively used to design strategies for determining the payoff in a conflict situation [3]. In this paper, we use the added value concept for designing an arbitration scheme that determines the payoff to each agent, based on certain *fair* norms.

When neutrality is implemented as a hard constraint, we have $\sum_m \Delta_m = \sum_n \Theta_n = R/2$. We will consider only the case of distribution of payoff to buyers as roles of buyers and sellers are symmetric and the same line of arguments hold true for sellers. Let the TDP has determined the total payoff $X(= R/2)$ to buyers $1, 2, \ldots, M$ with final trade $\alpha_1, \alpha_2, \ldots, \alpha_M$, respectively. Let X_{-m} be the total payoff determined by TDP when bid m is removed. It is worth noting here our assumption that a buyer places only one bid.

Definition 7 (Added Value). *Added value of a buyer (bid) m,* $V_m = X - X_{-m}$.

The above definition assumes that the game is only among the buyers unlike the original definition in [3], which also includes the sellers. This is mandatory to implement neutrality.

Proposition 1. $V_m \geq 0$ *for buyer m.*

Proof. Let $V_m < 0$. Then $X_{-m} > X$, which contradicts the fact that R $(X = R/2)$ is the maximum surplus of all the possible trades. □

We have implicitly assumed in the above proof that TDP is solved to optimality. If it is solved approximately, then it possible that $X_{-m} > X$. This assumption is equivalent to the unrestricted bargaining assumption of [2]

Proposition 2. *It need not always be true that* $\sum_m V_m = X$.

Proof. Consider a trade with total value X (to the buyers) and two buyers and one seller. Let the total trade $\alpha_1 + \alpha_2$ be aggregated and bought from the seller. If the seller is not interested to sell anything less than $\alpha_1 + \alpha_2$ then $V_1 = V_2 = X$. □

Thus the added value is not the payoff to the agents. The added-value measure provides a partial, rather than a complete, answer to the question of how value is divided [2]. It is used to develop business strategies to determine the payoffs. We use the added value to devise an arbitration scheme that determines the payoff to all the buyers.

5.2 Arbitration Scheme

When the players in a conflict situation are unable to come to a conclusion, they will submit their conflict to an external arbiter who will resolve the conflict based on certain *fairness* criteria that convinces all players [8]. Let $V = (V_1, V_2, \ldots., V_M)$ be the added values of the buyers. We have to determine $x = (x_1, x_2, \ldots, x_M)$ where x_m is the *proportion* of the total payoff X to be granted to buyer m.

Definition 8 (Arbitration Scheme). *An arbitration scheme is a function which associates to each conflict situation* $V \subseteq \mathbb{R}_+^M$ *a unique* $x \subseteq \mathbb{R}_+^M$ *to the buyers.*

There clearly exists an infinity of such functions. We have to choose one that is *fair* to all the players. The *fairness* criteria are certain intuitive relationship that V_i and x_i should satisfy. We frame these fairness criteria as axioms and mathematically investigate the existence of such an arbitration scheme.

Axiomatic Approach. We take an axiomatic approach in determining the arbitration scheme by specifying a set of basic axioms that need to be satisfied.

Axiom 1. $x_m \geq 0$*, and* $x_m = 0$ *iff* $V_m = 0$.

This is quite obvious as the proportions have to be non-negative and a buyer who has not added any value should get zero payoff i.e. he pays his bid price. In [2], x_m can be zero even if $V_m > 0$.

Axiom 2. $V_m > V_k \Leftrightarrow x_m > x_k$ *and* $x_m = x_k$ *iff* $V_m = V_k$.

The buyer with a higher added value should get more payoff and buyers with equal added values should get equal payoffs.

Axiom 3. *For any given* V_i, V_j, V_l, V_m *with* $V_l \neq V_m$, $\frac{V_i - V_j}{V_m - V_l} = \frac{x_i - x_j}{x_m - x_l}$.

This axiom says that the rate of change of payment should be constant with respect to that of the added value. This avoids any unfair scheme that might favor buyers with either high added value or low added value.

Axiom 4. $\sum_m x_m = 1$.

This ensures BB ($\sum_m x_m \not> 1$) and the payoff distribution is Pareto-efficient ($\sum_m x_m \not< 1$).

Proposition 3. *The arbitration scheme with* $x_m = \frac{V_m}{\sum_m V_m}$ $\forall m$ *is the unique closed form function that satisfies all the four axioms.*

Proof. Axiom 3 means that x_m varies linearly with V_m and so we have $x_m = a_m V_m + b_m$ for some real a_m and b_m. Axiom 2 states that x_m is an increasing function and all x_m have to be the same so that they are equal at the same values of V_m. Hence we have $x_m = aV_m + b$ $\forall m$ with $a > 0$. Since $x_m = 0$ at zero added value by Axiom 1, it is a linear function of V_m i.e. $x_m = aV_m$. By Axiom 4,

$$\begin{aligned} & \textstyle\sum_m x_m = 1 \\ \Rightarrow & \textstyle\sum_m aV_m = 1 \\ \Rightarrow & \ a = \frac{1}{\sum_m V_m} \\ \Rightarrow & \ x_m = \frac{V_m}{\sum_m V_m} \end{aligned}$$

□

Proposition 4. *The above arbitration scheme is IR.*

Proof. The payoff to m, $\Delta_m = x_m X = (\frac{V_m}{\sum_m V_m})X \geq 0$. □

The above arbitration scheme being the unique closed form function that satisfies all the axioms is worth noting, because with the existence of multiple functions, each player will favor the one that gives him more payoff [8]. However, it requires $(N + M + 1)$ TDPs to be solved, which may demand additional computational requirements if the TDP is hard to solve.

5.3 Limitations

In this section we discuss some limitations of using the above arbitration scheme using simple example exchanges. We consider a homogeneous exchange with bid structure (*Quantity*, *Unit Price*). The bids are all-or-nothing bids, i.e. either entire *Quantity* is traded or none is traded.

Equal Added Values. We will show an example where buyers get equal added values which is not fair. Consider a single seller with offer (100, 1) and two buyers A and B with bids (75, 2) and (25, 1.5), respectively. The optimal trade is that seller sells 75 units to buyer A and 25 units to buyer B. One can easily see that $X = 87.5/2 = 43.75$. Without A there will be no trade as the supply and demand are not matched and similarly without B there will be no trade. Hence, $V_A = V_B = 43.75$, $x_A = x_B = 0.5$, and payoffs $\Delta_A = \Delta_B = 21.875$. The two buyers are getting the same payoff even though one is buying only one-third of the goods at a much lower price. By the added value concepts this is perfectly fine as both buyers have equal bargaining power since they need each other to get a positive payoff. However, its use in the arbitration scheme fails to be fair.

Zero Added Values. Consider an exchange with two sellers X and Y with offers (75, 1) and (25, 1), respectively. Let there be three buyers: A with bid (50, 2), B with (50, 2) and C with (50, 2). Here we have three optimal allocations: X and Y trade with A and B, or with A and C, or with B and C. Let us choose the first one. Then, $V_A = V_B = 0$ as $X_{-A} = X_{-B} = X$ by the alternate optimal solution. We will have added values of the buyers as zero for any optimal solution. Again, by the added value concept this is fine, as buyers will have nearly zero value when the supply is limited [3]. But it is not fair in our case as the buyers gets zero payoff. In fact we cannot even apply our arbitration scheme in this case.

The equal added values case arise due to the all-or-nothing structure of the bids. One can expect more such pathological cases when we go for more complex combinatorial bid structures. We have to use the arbitration scheme with additional constraints so that we avoid such unfair situations. The zero added values case is due to the presence of alternate optimal solutions. This can be avoided by using the arbitration scheme only on the winning bids. Thus, we may have to impose more constraints on an exchange depending on its structure while using the arbitration scheme to avoid *unfair* results.

6 Conclusion

We proposed a new approach to the design of exchanges by decoupling allocation from pricing. Allocation is the trade determination problem that maximizes the reported surplus of the agents. The general approach to the pricing problem is to achieve IC or dominant strategies. We proposed a different approach, an arbitration scheme that determines the payoff to an agent based on relative value of the bid in the market. The payoff achieves pricing that is budget-balanced and individually-rational for all the agents. The arbitration scheme is based on the added-value concept of game theory and developed using an axiomatic approach which captures the intuitive relationships between the added-value and the payoff. The arbitration scheme has some limitations but can be used with certain additional constraints, which depends on the type of exchange that is being designed. This is a first step towards determining pricing through payoffs and work remains to be done to consider the computational complexity and equilibrium strategies of the agents.

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