COLOURED PETRI NET MODELS FOR AUTOMATED MANUFACTURING SYSTEMS

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ABSTRACT

In this paper, we propose an approach, using Coloured Petri Nets (CPN) for modelling flexible manufacturing systems. We illustrate our methodology for a Flexible Manufacturing Cell (FMC) with three machines and three robots. We also consider the analysis of the FMC for deadlocks using the invariant analysis of CPNs.

1. INTRODUCTION

A Flexible Manufacturing System (FMS) is an integrated, computer controlled configuration of machine tools and automated material handling devices that simultaneously process medium-sized volumes of a variety of part types. High productivity is achieved in such systems by effectively incorporating principles of Group Technology, Total Quality Control, etc., and following production control strategies such as Manufacturing Resource Planning (MRP-II) and Just-in-Time (JIT) production.

FMS is a discrete event dynamical system in which the workpieces of various job classes enter the system asynchronously and are processed concurrently, sharing the limited resources, viz., workstations, robots, MHSs, buffers and so on. Modelling, analysis and performance evaluation studies of FMSs are of immense practical interest to establish feasibility, evaluate qualitative and quantitative performance and compare alternative FMS configurations. In this paper, we establish the power of coloured Petri nets in modelling all the interactions in an FMS and also in establishing qualitative properties such as freedom from deadlocks, detection of specification errors, buffer overflows, etc.

2. A REVIEW OF MODELS FOR FMSs

2.1 Queueing Networks

One of the recent developments in analytical models of manufacturing systems has been the growing use of queueing network (QN) models for system planning and operation. Network-of-queues models take into account system dynamics, interactions and uncertainties inherent in FMSs. Also efficient computational algorithms are available for solving QN models. Solberg's[1] FMS model is a closed queueing system of exponential servers, with a single customer class. It mainly uses Buzen's algorithm[2]. Recently, Suri and Hildebrant[3] have proposed to view FMS as a closed QN with multiple customer classes and solved the design and real time operation problems using mean value analysis method. Notable contributions have also been made by Buzacott, Shanthikumar & Yao. Their contributions are summarised in[4,5,6].

2.2 Simulation

The modelling and performance analysis of FMS's can be conducted using discrete event simulation[7,8,9]. Such a simulation views the system operation as a succession of events, and, in principle, can mimic system behaviour in as much detail as is desired. However, performance optimization with respect to a number of decision parameters requires a large number of simulation runs. Also, because of the randomness involved in FMS operations such as breakdowns, queues, etc., multiple simulation runs and subsequent output analysis are required to reduce the simulation error. Thus, the method could be computationally very demanding.

2.3 Perturbation Analysis

Perturbation analysis (PA)[10] is another important technique useful in the performance analysis of FMS's. Using either the real-time data or that obtained from simulation, PA method finds the sensitivity of system performance measures to changes in decision parameters. For simulation studies of N-parameters, it provides N-fold increase in the efficiency over the "brute force" simulation method. PA typically answers questions such as "what would be the production of all types today if there were one more fixture of part type A"?.

2.4 Petri nets and Coloured Petri nets

Models of manufacturing systems based on Petri nets[11,12] and coloured Petri nets (CPNs)[13] have received wide attention recently. Petri nets and CPNs are graphical models and provide a compact framework for modelling and analysis of FMSs. The interactions in an FMS involve unpredictable and concurrent events and net-based models are very well suited for capturing these interactions. The mathematical theory of Petri nets and CPNs is very well developed and the theory of invariants, in particular, is very useful in the analysis and verification of a system modelled by nets. Some of the important properties of a system that can be investigated using Petri nets and CPNs are: liveness (absence of deadlocks), boundedness (finiteness of buffers and resources required), and properness (recoverability from failures). Besides the above qualitative properties, Petri nets and CPNs can also be used in designing a systematic simulation of FMSs. Further, by assigning times

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(deterministic or random) to the Petri net and CPN models, several performance measures (throughput rate, machine utilizations, effect of failures, etc.) can be computed. The application of ordinary Petri nets to FMSs is investigated in[9,14,15,16,17]. Significant contributions in applying CPNs to manufacturing systems are reported in[9,18,19,20].

3. INTRODUCTION TO COLOURED PETRI NETS

A Petri net is a directed bipartite graph composed of four parts: a set of places P, a set of transitions T, an input function I and an output function O. The input and output functions relate the transitions and places. Coloured Petri nets are extensions of Petri nets, with identical modelling description power but more concise from a graphical point of view. This conciseness is achieved by merging analogous elements in a model intosingle places and transitions and associating colours to tokens to distinguish among various elements. A transition can fire with respect to each of its colours.

Definition 1: A coloured Petri net (CPN) is a quintuple (P,T,C,I,O) where

(a) $P = \{p_1, p_2, \ldots, p_n\}$, $n \geqslant o$, is a set of places, $T = \{t_1, t_2, \ldots, t_m\}$, $m \geqslant o$, is a set of transitions such that $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$.

(b) C(p) and C(t) are the sets of colours associated with place $p \in P$ and transition $t \in T$. Also let

$$C(p_i) = \{a_{i1}, a_{i2}, \ldots, a_{iu_i}\} \; ; \; u_i = |C(p_i)| \; ; \; i = 1,2,\ldots,n.$$

$$C(t_j) = \{b_{j1}, b_{j2}, \ldots, b_{jv_j}\} \; ; \; v_j = |C(t_j)| \; ; \; j = 1,2,\ldots,m.$$

(c) $I(p,t)$: $C(p) \times C(t) \longrightarrow N$ is an input function where N is the set of all non-negative integers and
$O(p,t)$: $C(p) \times C(t) \longrightarrow N$ is an output function.

In this paper, we consider CPNs with finite number of places, finite number of transitions and finite colour sets.

Pictorially, places in CPNs are represented by circles and transitions by horizontal bars. If $I(p_i,t_j)\,(a_{ih}, b_{jk}) \neq 0$ for some h and k, then we draw a directed arc from place p_i to transition t_j. This directed arc is labelled with linear function $I(p_i,t_j)$. Similarly if $O(p_i,t_j)\,(a_{ih},b_{jk}) \neq 0$ for some h and k, we draw a directed arc from t_j to p_i and label it with $O(p_i,t_j)$.

Example 1. Consider a set of two machines m_1 and m_2 processing two part types J_1 and J_2. Each part type goes through one stage of operation and this operation can be performed on either m_1 or m_2. Fig.1 depicts the coloured Petri net model (CPNM) of this situation. The places, transitions, colours have the following interpretation.

p_1	: AJOBS	: Available Fresh Jobs
p_2	: AMACHS	: Available Machines
p_3	: PROC	: Processing in progress

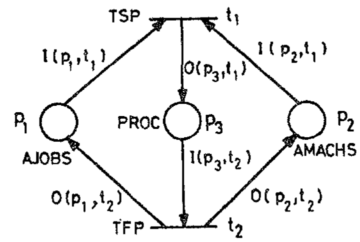

Fig.1. CPN model of a two machine system processing two part types (Example 1).

t_1	= TSP	: Transition indicating start of processing
t_2	= TFP	: Transition indicating finishing of processing

In the CPN model, we have three colour sets namely MACHS, PARTS and PARTS X MACHS where MACHS $= \{m_1, m_2\}$; PARTS $= \{J_1, J_2\}$; MACHS X PARTS $= \{(m_1,J_1), (m_1,J_2), (m_2,J_1), (m_2,J_2)\}$. Also C(AJOBS) = PARTS

C(AMACHS) = MACHS

C(PROC) = C(TFP) = C(TSP) = MACHS X PARTS

We can now define the input and output functions. First, we consider the input function of transition TSP with reference to its various colours and colour J_1 of place p_1.

$$I(\text{AJOBS, TSP})\,(J_1, (m_1,J_1)) = 1$$
$$I(\text{AJOBS, TSP})\,(J_1, (m_1,J_2)) = 0$$
$$I(\text{AJOBS, TSP})\,(J_1, (m_2,J_1)) = 1$$
$$I(\text{AJOBS, TSP})\,(J_1, (m_2,J_2)) = 0$$

Further,

$$I(\text{AJOBS, TSP})\,(J_2, (m_1,J_1)) = 0$$
$$I(\text{AJOBS, TSP})\,(J_2, (m_1,J_2)) = 1$$
$$I(\text{AJOBS, TSP})\,(J_2, (m_2,J_1)) = 0$$
$$I(\text{AJOBS, TSP})\,(J_2, (m_2,J_2)) = 1$$

More concisely, we can write down the input function defined by the $|C(p_1)| \times |C(t_1)|$ matrix $I(p_1,t_1)$ as follows:

$$I(p_1,t_1) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Similarly $I(p_2,t_1)$ is given by

$$I(p_2,t_1) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Now, it is easy to see that

$$O(p_3,t_1) = I(p_3,t_2) = I_4.$$

Similarly $O(p_1,t_2)$ and $O(p_2,t_2)$ are given by

$$O(p_1,t_2) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = I(p_1,t_1)$$

$$O(p_2,t_2) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = I(p_2,t_1)$$

Definition 2: A marking of a CPN is a function M defined on P such that for $p \in P$, $M(p)$: $C(p) \longrightarrow N$. A marked CPN is a CPN with a marking defined on it.

Let M_o and M denote the initial and current markings of a CPN. The markings are $(n \times 1)$ vectors with components $M_o(p_i)$ and $M(p_i)$ respectively where $M(p_i)$ represents marking of a place p_i. $M(p_i)$ is generally represented by formal sum of colours, i.e.

$$M(p_i) = \sum_{h=1}^{u_i} n_{ih} a_{ih}$$

where n_{ih} is the number of tokens of colour a_{ih} in place p_i.

Definition 3: A transition t_j of a CPN is said to be enabled with respect to a colour b_{jk} in a marking M if and only if

$$M(p_i)\ (a_{ih}) \geqslant I(p_i,t_j)\ (a_{ih},b_{jk}),\ \forall\, p_i \in P,\ a_{ih} \in C(p_i).$$

When a transition is enabled it can fire. When a transition t_j, enabled in a marking M, fires, with respect to a colour b_{jk}, a new marking M' is reached according to the following equation

$$M'(p_i)(a_{ih}) = M(p_i)(a_{ih}) + O(p_i,t_j)(a_{ih},b_{jk}) - I(p_i,t_j)(a_{ih},b_{jk}),\quad a_{ih} \in C(p_i);\ p_i \in P.$$

Example 2: Consider the CPN model shown in Fig.1. Suppose we start the process with one job of each part type and one machine of each type, then the initial marking is

$$M_o(p) = \begin{bmatrix} J_1 + J_2 \\ m_1 + m_2 \\ 0 \end{bmatrix}$$

In this marking the transition t_1 is enabled with respect to all its four colours. It can fire individually with respect to any one of the colours. Also it can fire concurrently with respect to colours (m_1,J_1) and (m_2,J_2) or (m_1,J_2) and (m_2,J_1). The markings reached in all these cases are given by

$$\begin{bmatrix} J_2 \\ m_2 \\ (m_1,j_1) \end{bmatrix};\ \begin{bmatrix} J_2 \\ m_1 \\ (m_2,J_1) \end{bmatrix};\ \begin{bmatrix} J_1 \\ m_1 \\ (m_2,J_2) \end{bmatrix};\ \begin{bmatrix} J_1 \\ m_2 \\ (m_1,J_2) \end{bmatrix};$$

$$\begin{bmatrix} 0 \\ 0 \\ (m_1,J_1) + (m_2,J_2) \end{bmatrix};\ \begin{bmatrix} 0 \\ 0 \\ (m_1,J_2) + (m_2,J_1) \end{bmatrix}$$

Definition 4: The incidence matrix W of a CPN is a $\sum_{i=1}^{n} u_i \times \sum_{j=1}^{m} v_j$ matrix. It is defined by the $n \times m$ block matrix

$$W = \left[w_{ij}\right] \quad \begin{array}{l} i = 1,2,\ldots,n \\ j = 1,2,\ldots,m \end{array} \quad \text{where } w_{ij} = O(p_i,t_j) - I(p_i,t_j)$$

Definition 5: A weighted set of transitions in a CPN is a function x defined on T such that

$$x(t): C(t) \longrightarrow N \quad \forall\ t \in T$$

Let M be any marking reachable from M_o. Then, it follows from definition 4 that there exists a non-negative weighted set of transitions x such that

$$M = M_o + W.x \tag{3.1}$$

Definition 6: Let Q be a non-empty set of colours, of cardinality q. A weighted set of places, with respect to Q, is a function X defined on P such that

$$X(p): Q \times C(p) \longrightarrow Z\ ,\quad Z \text{ being the set of integers.}$$

A weighted set of places may also be represented by a block matrix $X = [x_1, x_2, \ldots, x_n]$ where x_i, $i = 1,2,\ldots,n$, is a matrix of integers of dimension $q \times u_i$.

Definition 7: A weighted set of places $X = [x_1, x_2,\ldots,x_n]$ is called a place-invariant or p-invariant of a CPN (P,T,C,I,O) with respect to a non-empty set Q iff $XW = 0$ (3.2)

ie., $\sum_{i=1}^{n} x_i \cdot w_{ij} = 0$ for each $j=1,2,\ldots,m$,

where $\sum$ is with respect to matrix addition, '.' denotes matrix multiplication, and 0 is the zero-matrix of dimension $q \times v_j$.

Let M be any marking reachable from M_o. Then, using (3.1) and (3.2), we get

$$XM = XM_o\ \forall M \in R[M_o] \tag{3.3}$$

Thus if X is a p-invariant, the weighted product XM remains invariant for all reachable markings of CPN. This is an important property of a p-invariant.

Definition 8: A marked CPN (P,T,C,I,O,M_o) is said to be deadlock-free iff there does not exist a marking $M_d \in R[M_o]$ in which all transitions are disabled w.r.t. all possible colours. Such a marking M_d, if it exists, is called a deadlock.

The p-invariants of a CPN model are very useful in the investigation of presence/absence of deadlocks in the CPN.

4. A MANUFACTURING CELL WITH MULTIPLE ROBOTS

In this section, we consider a flexible manufacturing cell comprising three workstations m_1,m_2, and m_3, three robots r_1,r_2, and r_3, and a load/unload area. This cell is pictured in fig.2. We construct a CPN model of the FMC and show the presence of a deadlock under certain operating policies. In fig.2, m_1,m_2, and m_3 represent three machines in the cell and r_1,r_2, and r_3 represent three robots. Robot r_1 is used for loading and unloading of parts and for part and tool transfer between m_1 and m_3. Robots r_2 and r_3 are used for part and tool transfers between machines m_1,m_2 and m_2,m_3 respectively. It is assumed that the tools in the system are of three types t_1,t_2, and t_3 and that a tool of type t_i (i=1,2,3) is present at machine m_i. We also assume that there is a perennial supply of raw workpieces.

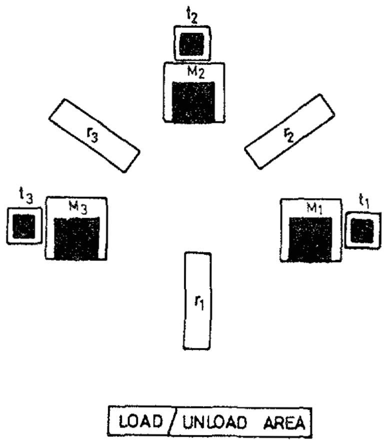

Fig.2. An automated manufacturing cell with three machines and three robots.

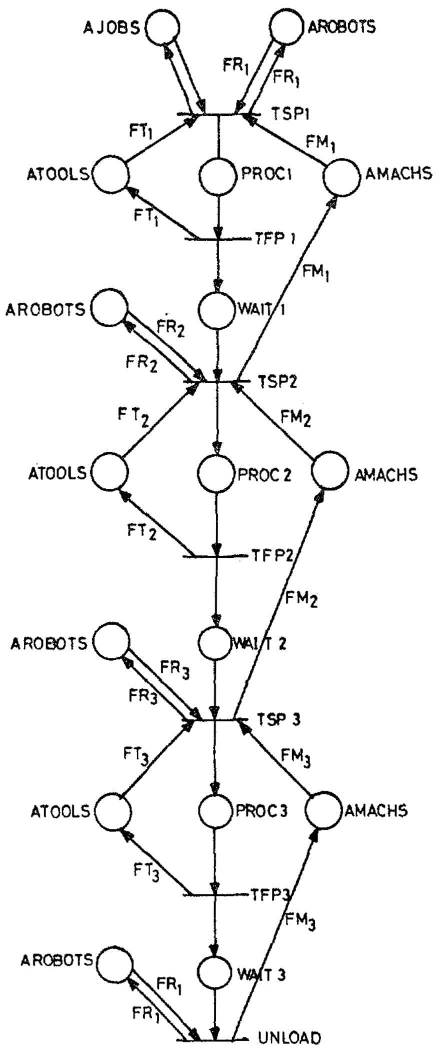

Fig.3. CPN model of an FMC with three machines, three robots and two part types.

We consider the processing of two part types J_1, J_2 with the following sequence of operations.

$$J_1: \quad m_1\,(t_1) \rightarrow m_2(t_2) \rightarrow m_1\,(t_1)$$
$$J_2: \quad m_3\,(t_3) \rightarrow m_2\,(t_2) \rightarrow m_3\,(t_3)$$

Each part type undergoes three stages of operations. For example, a part of type J_1 is first processed by m_1 using a tool of type t_1, then by m_2 using a tool of type t_2, and finally by m_1 using a tool of type t_1. We assume that r_1 loads a part into the system as soon as the machine meant for its first stage of operation becomes free (Idle machine rule).

Fig.3 shows a CPN model that captures all interactions in the above system.

We define the following colour sets

$$\text{MACHS} = \{m_1, m_2, m_3\}\ ;\ \text{ROBOTS} = \{r_1, r_2, r_3\}$$
$$\text{TOOLS} = \{t_1, t_2, t_3\}\ ;\ \text{JOBS} = \{J_1, J_2\}$$

The interpretation of the places and transitions and their colour sets is given in table 1.

Place/ transition	Interpretation	Colour set
AJOBS	Available raw parts	JOBS
AROBOTS	Available robots	ROBOTS
AMACHS	Available machines	MACHS
ATOOLS	Available tools	TOOLS
PROC 1	First stage of processing	JOBS
WAIT 1	Waiting for machine after first stage of processing	JOBS
PROC 2	Second stage of processing	JOBS
WAIT 2	Waiting for machine after second stage of processing	JOBS
PROC 3	Third stage of processing	JOBS
WAIT 3	Waiting for robot to get unloaded	JOBS
TSP 1	First stage starts	JOBS
TFP 1	First stage finishes	JOBS
TSP 2	Second stage starts	JOBS
TFP 2	Second stage finishes	JOBS
TSP 3	Third stage starts	JOBS
TFP 3	Third stage finishes	JOBS
UNLOAD	A finished part gets unloaded	JOBS

Table 1. Places, transitions, their interpretation and colour sets, for the CPN model of fig.3.

The functions involved in the above CPN model are

$$FM_1, FM_2, FM_3 \ :\ \text{MACHS X JOBS} \rightarrow N$$
$$FT_1, FT_2, FT_3 \ :\ \text{TOOLS X JOBS} \rightarrow N$$
$$FR_1 FR_2, FR_3, FR_4 \ :\ \text{ROBOTS X JOBS} \rightarrow N$$

In matrix notation, these functions are:

$$FM_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\ ;\ FM_2 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}\ ;\ FM_3 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = FM_1$$

$$FT_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} ; \; FT_2 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} ; \; FT_3 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$FR_1 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} ; \; FR_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} ;$$

$$FR_3 = FR_2 \; ; \; FR_4 = FR_1$$

It is to be noted that the weights of the unlabelled arcs in the CPN model of fig.3 are identify matrices of appropriate orders.

We shall assume the initial marking M_0 of the CPN model to be : all machines free, all robots free, all tools free, and a fresh part of each type available in the load/unload area. Thus we have

M_o (AJOBS) = $J_1 + J_2$; M_o (AMACHS) = $m_1+m_2+m_3$;
M_o(AROBOTS) = $r_1+r_2+r_3$; M_o (ATOOLS) = $t_1+t_2+t_3$;
M_o (PROC 1) = M_o (PROC 2) = M_o (PROC 3) = 0

M_o (WAIT 1) = M_o (WAIT 2) = M_o (WAIT 3) = 0

Table 2 shows two p-invariants I_1 and I_2 of this model. I_1 is w.r.t. the colour set MACHS and I_2 is w.r.t. the colour set TOOLS. These invariants can be obtained using the results of [21].

Table 2. Place invariants of the CPN model of fig.3.

Place	components of Invariant I_1	I_2
AJOBS	ID	ID
AROBOTS	ID	ID
AMACHS	ID	0
ATOOLS	0	ID
PROC 1	FM_1	FT_1
PROC 2	FM_2	FT_2
PROC 3	FM_3	FT_3
WAIT 1	FM_1	0
WAIT 2	FM_2	0
WAIT 3	FM_3	0

The interpretation of I_1 and I_2 may be obtained using equation 3.3. Substituting $X = I_1$ in (3.3), we get for any reachable marking $M \in R[M_0]$,

$$M(AMACHS) + FM_1 . M(PROC\ 1) + FM_2 . M(PROC\ 2) + FM_3 . M(PROC\ 3) + FM_1 . M(WAIT\ 1) + FM_2 . M(WAIT\ 2) + FM_3 . M(WAIT\ 3) = m_1+m_2+m_3 \tag{4.1}$$

Equation (4.1) shows that the machines in the cell are either idle or processing a part or blocked.

Using equation (3.3) and substituting $X=I_2$, we obtain

$$M(ATOOLS) + FT_1 . M(PROC\ 1) + FT_2 . M(PROC\ 2) + FT_3 . M(PROC\ 3) = t_1+t_2+t_3 \tag{4.2}$$

Equation (4.2) implies that the tools are available or are being used insome stage of processing.

We now show the presence of a deadlock in the above system by considering the following sequence of events: (1) r_1 loads a part of type J_1 into m_1 and m_1 starts processing this part. Let us call this part p_1. (2). r_1 loads a part of type J_2 into m_3 and m_3 starts processing it. Let us call this part p_2. (3). m_1 finishes processing p_1, r_2 transfers p_1 from m_1 to m_2, and m_2 starts the second stage of operation on p_1. (4). r_1 loads a part of type J_1 into the system and m_1 starts processing this part, which we call p_3. (5). m_3 finishes first stage of operation on p_2. (6). m_1 finishes first stage of operation on p_3. (7). m_2 finishes second stage of operation on p_1.

Let us call the marking of the CPN model at this juncture as M. Then, we see that

M(WAIT 1) = $J_1 + J_2$ and M(WAIT 2) = J_1

Using the above in (4.1), we complete the marking M as

M(AMACHS) = 0 ; M(PROC 1) = 0 ; M(PROC 2) = 0

M(PROC 1) = 0 ; M (WAIT 3) = 0 ; M(AJOBS) = J_1+J_2;
M(AROBOTS) = $r_1+r_2+r_3$; M(ATOOLS) = $t_1+t_2+t_3$

Now, M(AMACHS) = 0 $\Longrightarrow$ transitions TSP1, TSP2, and TSP3 are disabled w.r.t. all of their colours.

M(PROC 1) = M(PROC 2) = M(PROC 3) = 0 $\Rightarrow$ transitions TFP1, TFP2, and TFP3 are disabled w.r.t. all colours.

M(WAIT 3) = 0 $\Longrightarrow$ transition UNLOAD is disabled w.r.t. all colours.

Thus in the above marking M, all transitions in the CPN model are disabled w.r.t. all their colours and hence M is a deadlock. It can be observed that this deadlock has occured because of the flow control policy which causes a fresh part to be loaded into the system whenever the machine that can do the first stage of processing on that part is available. If the flow control policy is such that a fresh part of a particular type is loaded into the system only when no unfinished part of that type is in the system, this deadlock can be avoided. Alternatively, by having a buffer to store all waiting parts, this deadlock can be avoided.

5. CONCLUDING REMARKS

In this paper, we have established the usefulness of coloured Petri nets as a compact modelling tool for the description of automated manufacturing systems. By constructing the CPN model of a flexible manufacturing cell, we have shown the modelling power of CPNs and their use in deadlock investigation. Elsewhere[22], we have presented CPN models for (i) a flexible assembly cell with three machines and three robots, (ii) an FMS with operation flexibility, and (iii) a generalized FMS. We have also shown the analysis of these systems for deadlocks using the CPN invariants. We are unable to include these examples owing to space constraints and the interested readers may obtain a copy of the technical report[22] from the first author.

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