Non-stationary Models of Manufacturing Systems: Relevance and Analysis

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Abstract

Performance evaluation studies in manufacturing systems have traditionally considered models in which the arrival process and service process are time independent. Real-world manufacturing systems however, are subjected to highly complex and usually time-dependent input workloads. This motivates the study of performance models of manufacturing systems under non-stationary conditions. In this paper, we present several situations in manufacturing systems where non-stationary models are relevant. For studying such models, transient analysis is more appropriate than steady-state analysis. We explore various techniques for analyzing such models, including numerical and simulation techniques, and present two illustrative examples.

1 Introduction

Studies in performance evaluation of discrete manufacturing systems, using simualtion or analytical models have always emphasized steady-state or equilibrium performance in preference to transient performance. Transient analysis is inevitable in the case of manufacturing system models that do not attain a steady state or equilibrium [1, 2]. Even in models that do attain an equilibrium, transient analysis is important for studying performance over finite intervals, particularly in the early phase of evolution of the system. This paper is concerned with the transient analysis of non-stationary models of manufacturing systems, *i.e.*, models in which the arrivals are time dependent or the service process is time dependent or both.

The motivation for this paper is the following. Traditional stochastic models of manufacturing systems [3, 1] have only considered homogeneous arrival processes (*e.g.*, homogeneous Poisson process, determinstic or periodic arrival sequences) and time-independent service rates. However, a real-world manufacturing system usually operates under non-stationary conditions because of fluctuating demands for products, randomness involved in raw material availability, *etc.* Thus the workloads offered to a manufacturing system are time dependent or non-stationary. Only transient analysis is relevant for such models.

Performance studies of non-stationary models appear in the literature, mostly in the context of congestion modeling in data networks. Tipper and Sundareshan [4] have explored numerical methods for analyzing computer network models under non-stationary conditions. Lovegrove, Hammond, andTipper [5] discuss simulation methods for the same problem. Duda [6] has investigated diffusion approximation methods for conducting transient analysis of non-stationary queueing systems. Van As [7] has studied several non-stationary, Markovian queueing systems by formulating and numerically solving coupled differential equations using the fourth-order Runge-Kutta method. Fischer and Meier-Hellstern [8] have recently surveyed many results concerning models that use the Markov modulated Poisson process which can be used for modeling non-homogeneous Poisson arrival process. More recently, non-stationary models of congestion effects of link failures in wide area networks have been investigated by Tipper *et al.* [5]. A survey of transient analysis of manufacturing systems appears in [2].

The paper is organized as follows. In Section 2, we briefly survey the traditional arrival processes that have been considered in the manufacturing systems modeling literature. We then explore different possible models of non-stationary arrivals in the manufacturing systems. In Section 3, we discuss several methods for analyzing non-stationary models. Finally in Section 4, we present two simple examples of non-stationary queueing models of manufacturing systems.

2 Arrival Processes in Manufacturing System Models

2.1 Homogeneous Arrival Processes

Traditionally arrival processes have been assumed to be time independent. The following are some of the prominent arrival models.

2.1.1 Deterministic/Periodic Arrivals

Here raw parts are assumed to arrive at predetermined instants of time either periodically or aperiodically. If the service times are also deterministic, the analysis of the performance models can often be carried out easily for simple models. However analysis of complex models leads to combinatorial explosion. Such models can often be used to gain insight into system characteristics, such as for example, stability of queueing network models of manufacturing systems (see [9]). A variation of the deterministic arrival model that allows for some burstiness is discussed in [9]: in every time interval $[s, t]$, the number of arrivals in $[s, t] \leq \lambda(t - s) + \gamma$ where λ is a constant (arrival rate) and γ is another constant that allows for burstiness.

2.1.2 Constant WIP Arrivals

This arrival model is extensively used in all closed queueing network formulations of manufacturing systems [10, 1] by assuming that the population of customers in the network is constant. The justification for the assumption is provided by the availability of a fixed number of fixtures or pallets for carrying work inside the factory floor. The advantage of this model is the tractibility it provides to the analysis.

2.1.3 Poisson Process

In open queueing models of manufacturing systems [3, 1] and in most simulation studies, an arrival model that is invariably employed is the homogeneous Poisson process (*i.e.* constant mean arrival rate). Poisson arrivals lead to tractability of queueing models and are found to be quite adequate in many modeling situations.

2.1.4 Renewal Process

This is a more general arrival process than the Poisson process and makes it possibile to model non-exponential interarrival times. Many researchers have used it in their performance studies [3].
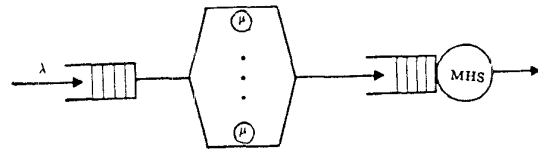

Figure 1: An example of Markov-modulated Poisson arrivals

2.2 Non-stationary Arrival Processes

In this section, we propose several different ways of modeling time dependent arrivals into a manufacturing system.

2.2.1 Markov-Modulated Poisson Process

A Markov-modulated Poisson process (MMPP) [8] qualitatively models time-varying correlations between the interarrival times while still remaining analytically tractable. It has been prominently used in the analysis of communication networks. An MMPP is a doubly stochastic Poisson process whose arrival rate is given by $\lambda[J(t)]$, where $J(t)$, $t \geq 0$, is a finite (say, m) state, irreducible Markov process. An MMPP may be constructed by varying the arrival rate of a Poisson process according to an m-state irreducible Markov chain which is independent of the arrival process. When the Markov chain is in state i, arrivals occur according to a Poisson process with rate λ_i.

As an example, consider a material handling system that transports finished parts from a pool of m machines to some destination (say, an automated storage and retrieval system)– see Figure 1. Assume that raw parts arrive into the system according to a Poisson process with rate λ and go through an operation on any of the available machines. If the processing time on any machine is exponentially distributed with mean $(1/\mu)$ and $\lambda < m\mu$, then the departure process from the pool of machines is Poisson with rate λ [1]. If the machines are unreliable and we assume additionally that the failure and repair times are much larger compared to inter-arrival and service times, the departure process from the pool of machines (*i.e.*, the arrival process into the MHS) can be modeled as an MMPP. In fact, the above system can be analyzed using the results for an MMPP/G/1 queue [8].
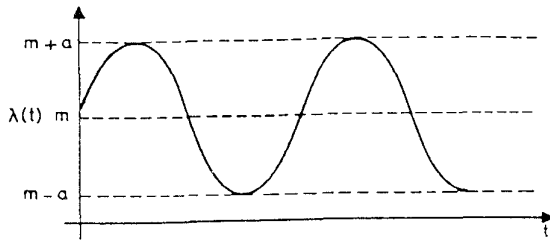
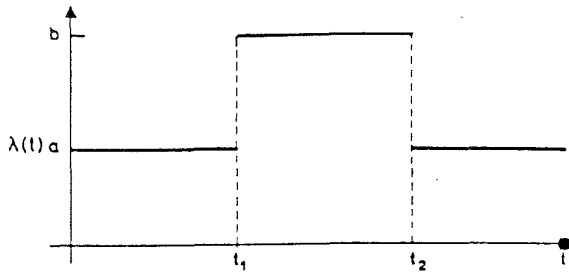Figure 2: A periodic Poisson arrival process

Figure 4: An arbitrary time-dependent arrival process

Figure 3: An arrival process with a burst

2.2.2 Non-homogeneous Poisson Process

In a non-homogeneous Poisson process, the arrival rate $\lambda(t)$ is a positive bounded function of t and such a process allows modeling of very general time-dependent arrivals. If $X(t)$ is a non-homogeneous Poisson process with $X(0) = 0$, then

$$P\{X(t) = k\} = \frac{e^{-\Lambda(t)}[\Lambda(t)]^k}{k!}, \quad \text{where}$$

$$\Lambda(t) = \int_0^t \lambda(s)ds$$

Figures 2, 3, and 4 show three different $\lambda(t)$ functions. In Figure 2, we have $\lambda(t) = m + a \sin(bt + c)$, where a, b, c, and m are positive real constants and $m \geq a$. The intensity of the Poisson process here varies from $(m - a)$ to $(m + a)$ in a sinusoidal fashion and this could be used to model waxing and waning demands for products in a manufacturing system. In Figure 3, the variation of $\lambda(t)$ is given by

$$\begin{aligned}
\lambda(t) &= a & (0 < t < t_1) \\
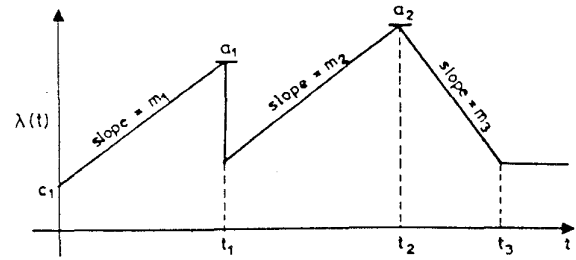&= b & (t_1 \leq t < t_2) \\
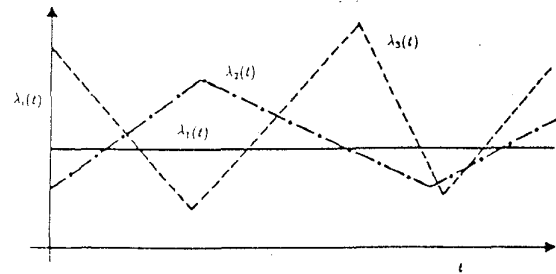&= a & (t_2 \leq t)
\end{aligned}$$

Figure 5: Non-homogeneous arrivals in a multiproduct context

The above can be used to model a sudden burst or a rush hour in the input traffic. The burst duration is $(t_2 - t_1)$, and the value of b indicates the peak traffic rate during the rush hour. Figure 4 shows an arrival pattern that gradually reaches a peak, then declines suddenly, next picks up again, and finally declines gradually and settles down at some level.

In general, $\lambda(t)$ can be suitably defined to model the arrival pattern as exactly as desired.

2.2.3 Arrival Patterns for Multiproduct Systems

In a multiproduct manufacturing system, the inputs for different products types could be different. Figure 5 shows an example where there are three product types and the individual arrival processes are non-homogeneous Poisson processes with rates $\lambda_1(t)$, $\lambda_2(t)$, and $\lambda_3(t)$, respectively. Product 1 can be viewed as having a steady demand while products 2 and 3 have fluctuating demands. Note that $\lambda_2(t)$ and $\lambda_3(t)$ have approximately complementary and in-phase variations.

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3 Analysis of Models under Non-stationary Conditions

With non-stationary models, transient analysis is more important than a steady-state anaysis as in most cases a steady state may not even exist. However, computation of time dependent or transient solutions is quite complex even in the case of a simple finite capacity M/M/1/K queueing model with a non-homogenous Poisson arrival process having (time varying) average arrival rate $\lambda(t)$. For the sake of generality, assume service rates as $\mu(t)$. If $p_n(t)$ for $0 \leq n \leq K$ is the probability of n customers in the system at time t, then the Chapman-Kolmogorov equations describing the evolution of these probabilities are [11, 4]:

$$\frac{dp_o(t)}{dt} = -\lambda(t)p_o(t) + \mu(t)p_1(t) \tag{1}$$

$$\frac{dp_n(t)}{dt} = \lambda(t)p_{n-1}(t) - [\lambda(t) + \mu(t)]p_n(t)$$
$$+\mu(t)p_{n+1}(t), \ \forall \ 0 < n < K \tag{2}$$

$$\frac{dp_K(t)}{dt} = \lambda(t)p_{K-1}(t) - \mu(t)p_K(t) \tag{3}$$

The above set of differential equations is notoriously difficult to solve analytically due to the time-varying coeffients [4]. Even when the arrival and service rates are constant and a unique steady-state equilibrium exists, the solution for the time-dependent state probabilities above involves an infinte sum of Bessel functions [11] and hence is computationally hard. In the general non-stationary case, the solution is even harder, even assuming $\lambda(t)$ and $\mu(t)$ to be smooth, well-behaved functions.

From the literature, one can isolate four prominent techniques for computing time-dependent performance measures under non-stationary conditions. These are discussed below.

3.1 Numerical Solution of Differential Equations

This method is discussed by Tipper and Sundareshan [4], Van As [7], and Odoni and Roth [12]. The basic idea is to approximate the time varying arrival and service rates by constants over small intervals and then numerically solve equations (1), (2), and (3) above, over all time intervals of interest. Numerical studies have shown that the fourth- or fifth- order Runge-Kutta method provides an efficient techinque in terms of efficiency and computing time [7]. This technique

is suitable only for small sized systems. For large systems and for large time intervals, the method is intractable.

3.2 Diffusion Approximations

Duda [6] has discussed a technique for conducting transient analysis of queueing systems under non-stationary traffic conditions, using diffusion approximations. The technique relies on heavy traffic limit theorems that enable GI/G/1 type of queueing systems to be approximated by a one dimensional reflected Brownian motion [13]. The solution of a partial differential diffusion equation subject to a boundary condition known as the elementary return barrier [14] can then be used to obtain transient solutions under all traffic conditions for a GI/G/1 queue. Duda [6] has used this approximation in the transient analysis of general queueing networks under non-stationary conditions, using parametric decomposition.

3.3 Nonlinear Differential Equations

In this approach, the time-dependent behaviour of a queueing network is approximated by a set of nonlinear differential equations describing the time-varying behaviour of certain mean performance measures such as queue lengths [4]. The resulting equations are solved using numerical methods. The approach can also be used for applying optimal control methods in designing systems based on transient measures [4].

3.4 Simulation Methods

The techniques mentioned above are tractable only for reasonably small sized systems and are efficient only under a Markovian setting. Simulation methods provide an attractive alternative when analytical or numerical methods are not feasible [15]. The simulation methods for non-stationary systems are different from the traditional steady-state simulation methods. Since time-dependent behaviour is sought, the simulation involves generating an ensemble of replications and computing performance measures as ensemble quantities [15]. Lovegrove, Hammond, and Tipper [15] have discussed in detail the various issues involved in the simulation of non-stationary behaviour.

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4 Examples

4.1 A Machine Subjected to a Demand Burst

Consider a single machine center producing a single type of product. An M/M/1 queue is the simplest model of such a system, assuming infinite buffer space. It is often the case that the demand for a product suddenly rises dramatically during a time interval. If infinite amount of raw material is available, it would be of interest to know how the system would respond to such a rush for the product. The arrival process in this case can be modeled as shown in Figure 3. Such an example in the context of data networks is discussed by Lovegrove, Hammond, and Tipper [15].

Let the mean service time for each product be 1 minute and $\lambda_{in}(t)$, the mean arrival rate of demands for the product, be given by

$$\begin{aligned}\lambda_{in}(t) &= 0.5/\text{min} \quad 0 \le t < 25 \\ &= 1.5/\text{min} \quad 25 \le t < 50 \\ &= 0.5/\text{min} \quad t \ge 50\end{aligned}$$

The transient performance measures of interest here would be [15] :

- $N(t)$, the ensemble average number in the system at time t,

- Derivative of the ensemble average number in the system at time t, i.e. $\frac{dN(t)}{dt}$.

- $\lambda_{out}(t)$, the ensemble average throughput rate of the system.

Figure 6 depicts $N(t)$, $\frac{dN(t)}{d(t)}$, and $\lambda_{out}(t)$ as computed using 1000 simulation replications [15]. The results can be used to fix the buffer sizes and estimate the delays incurred because of the sudden burst of demands for the product.

4.2 A Three Machine System Subjected to Periodic Poisson Input

Figure 7 shows a three machine queueing network that produces a product that involves two operations. The first operation is performed on M_1 (processing time is 10 minutes) while the second operation may be carried out on machine M_2 or M_3 (processing time is 20 minutes in either case). Among the parts that have finished their operation on M_1, 60% are routed to M_2 while the rest are routed to M_3. The input to the system is a non-homogeneous Poisson process with $\lambda(t)$
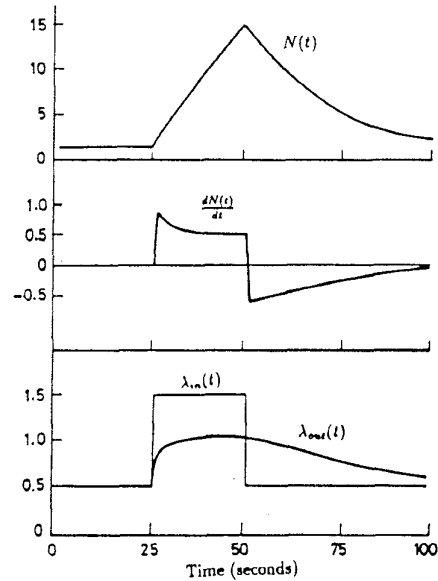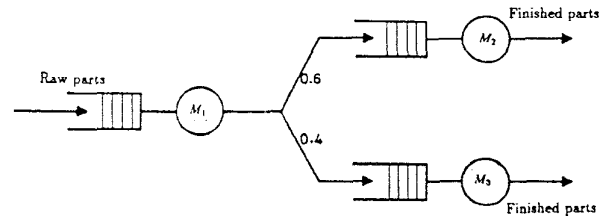

Figure 6: Transient performance measures

Figure 7: A three machine queueing network model

given by $\lambda(t) = 0.06\{1 + cos\left(\pi + \frac{2\pi t}{24}\right)\}$. This example is adapted from [6]. The input process here models the waxing and waning of demands or raw material arrivals. The above system is again non-stationary and it is of interest to study the time-dependent behaviour of the system.

Figure 8 shows the variation of time-dependent mean queue length at machines 1, 2, and 3 respectively. These results have been obtained by simulation [6]. The results can be used to predict the maximum and minimum cycle times and to estimate the buffer capacities required.

5 Conclusions

Most stochastic models of manufacturing systems consider the arrival and service processes involved there to be time independent and hence are poor candidates to
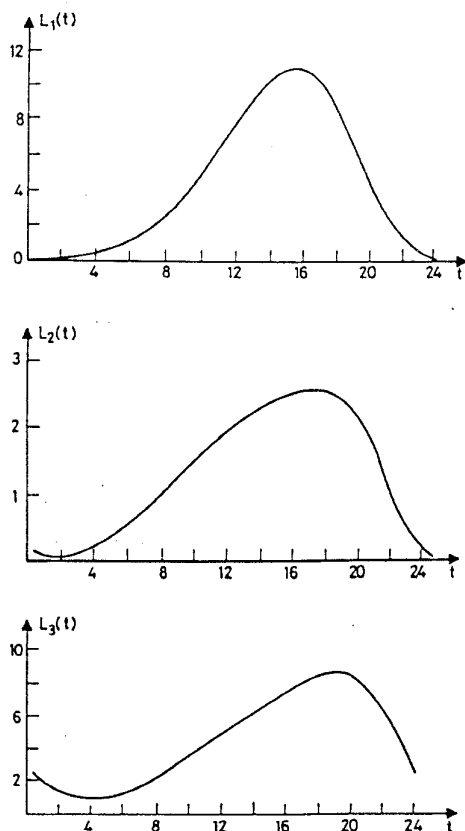
Figure 8: Time-dependent queue lengths at the individual queues

take into account the fluctuating demands *etc.* Nonstationary stochastic models can accommodate such realistic behaviour. Here, we presented some nonstationary models including Markov modulated Poisson process, non-homogeneous Poisson process, *etc.* We also discussed some available methodologies to analyze them and these include diffusion approximations, numerical solutions, and simulation. Further work includes coming up with better models and solution techniques thereof to gain better insight into the behaviour of non-stationary manufacturing systems.

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