The Core and Shapley Value Analysis for Cooperative Formation of Procurement Networks

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Abstract

Formation of high value procurement networks involves a bottom-up assembly of complex production, assembly, and exchange relationships through supplier selection and contracting decisions, where suppliers are intelligent and rational agents who act strategically. In this paper we address the problem of forming procurement networks for items with value adding stages that are linearly arranged. We model the problem of Procurement Network Formation (PNF) for multiple units of a single item as a cooperative game where agents cooperate to form a surplus maximizing procurement network and then share the surplus in a stable and fair manner. We first investigate the stability of such networks by examining the conditions under which the core of the game is non-empty. We then present a protocol, based on the extensive form game realization of the core, for forming such networks so that the resulting network is stable. We also mention a key result when the Shapley value is applied as a solution concept.

1. Introduction

To motivate the procurement network formation problem, which has recently emerged as a key problem within supply chain management, consider the following stylized scenario from the automotive industry: An automotive assembler, hereafter called the buyer, is interested in procuring stampings for assembly in an automobile. The supply chain for an automotive stamping typically spans many tiers. The buyer values the item at a certain price. The stamping undergoes many processes before it can be delivered to the car assembler for assembly in a car. Starting from the master coil, it undergoes cold rolling, pickling, slitting, and stamping. We assume that all these manufacturing operations are organized linearly and precedence constraints apply to the way in which the operations can be carried out as in Figure 1.

Now, a wide variety of suppliers with varying capabilities may be available in the market to meet the requirements of the buyer. That is, there may be suppliers who are only capable of doing cold rolling while there may be others who can do cold rolling, pickling, and slitting. There may still be others who can deliver the finished stamping by carrying out all the operations. In addition, between each of these manufacturing processes, the item may also have to be transported from one supplier location to another. Each of these suppliers incurs costs to carry out the processing at various stages of value addition. It is safe to assume that these costs vary from firm to firm.

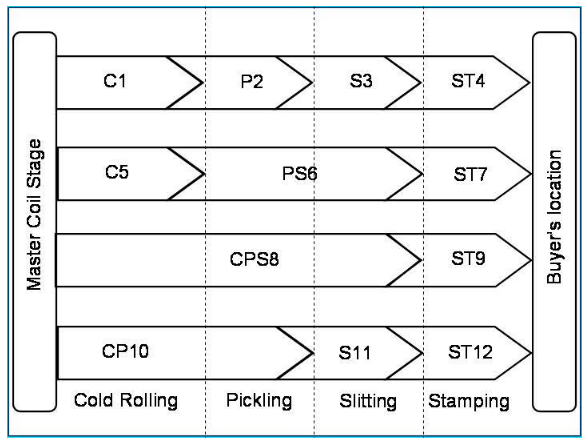

Figure 1. A linear supply chain for automotive stampings

Given all these different options in which the stamping can be procured, the decision of what would be the best (least cost) combination of suppliers to carry out the various operations is to be made. This can be done by constructing what we call the *procurement feasibility graph* which is shown in Figure 2.

In this graph, each edge represents a specific value adding operation. An edge is assumed to be owned by a supplier, hereafter called an agent. The fact that each agent incurs a certain cost of processing is captured as the cost of allowing a unit amount of flow on the edge owned by the agent. Also, the processing capacity of the supplier for a

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particular operation is indicated by an upper bound on the flow that is possible along the edge representing that supplier's operation. In Figure 2, we capture this information as a 3-tuple beside every edge in the graph. Further, we assume that there are two special nodes in the graph called the origin node and the terminal node that represent the starting point and ending point, respectively for all value adding operations.

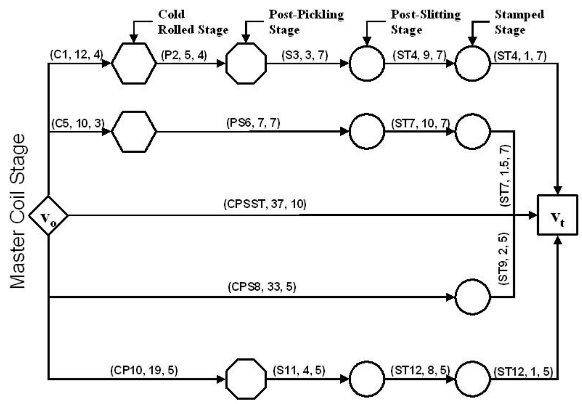

Figure 2. A procurement feasibility graph

A path from the origin node to the terminal node in the graph indicates one possible way of procuring one unit of the stamping. The cost of this procurement is obtained by summing the costs of the individual edges in that path. More generally, a flow of f units along a path costs f times the cost of that path. We are then naturally interested in finding a minimum cost way of providing the stampings to meet the buyer's demand or in other words finding a minimum cost flow through the network. This immediately indicates that the objective is to maximize the surplus value that could be generated by picking a combination of suppliers to service the buyer's requirements. Here we define surplus value as the difference between buyer's valuation times the amount of flow through the network and the cost of maintaining that flow. This is nothing but a surplus maximizing flow problem which we call the *single item, multi-unit procurement network formation (MPNF) problem*. This is the central problem that we explore in this paper.

1.1 Procurement Network Formation: How?

Two broad approaches have been used to study the procurement network formation problem when agents are assumed to act strategically to further their own payoffs. The first uses techniques based on competitive equilibrium analysis [16] and the second makes use of auction technology [1, 2, 8, 9]. While these approaches has been fruitful in advancing the state-of-the-art for some areas of applications within business-to-business transactions, practitioners have been reluctant to embrace purely auction based methods across the board [3]. Secondly, supply chain practitioners and researchers have long argued that a supply chain that is created is sustained when it is able to create a surplus value and the surplus is shared in a fair manner with the supply chain partners. These two points when taken together imply that models for procurement network formation should explicitly include the aspect of surplus sharing. Problems in surplus sharing and cost sharing are replete with ideas from *cooperative game theory*.

Cooperative game theory has two central themes. The first theme is that of *feasible outcomes* and the second is that of *stability*. These two central themes of cooperative game theory have their perfect analogs in the context of procurement network formation. In procurement network formation, we are interested in constructing the feasible set of outcomes and picking the *best* from among them. In addition, as argued by management scientists, supply chains remain stable only when each of the agents gets a fair share of the surplus that the supply chain generates. So, we are interested in finding a fair way of dividing the gains that accrue when agents come together to create surplus value. In doing so, we are interested in finding a stable coalition of partners. Clearly then, cooperative game theory provides us with an apt toolkit to study the procurement network formation problem. In fact, several recent survey articles have identified cooperative game theory as an important tool in the supply chain researchers toolkit [6, 3, 14, 17, 11, 4, 12].

1.2 Research Contributions and Outline of the Paper

In this paper, we focus on the MPNF problem when the agents who own edges in the procurement feasibility graph are completely aware of (1) each others' costs and (2) the maximum demand of the buyer and his valuation. Our specific contributions and the sequence in which we present them are as follows:

In Section 2, we model the MPNF problem as a cooperative game with complete information. In Section 3, we formalize through analytical results (a) the effect of edge ownership structure on the non-emptiness of the core, (b) the effect of the demanded quantity on the non-emptiness of the core, and (c) the effect of the buyer's valuation on the non-emptiness of the core of a special case of the MPNF game. In Sections 4 and 5, we present some of the other key results we have developed for the MPNF problem. Because of constraints of space we simply mention the results without proofs and other details. The detailed proofs could be looked up from [7]. In Section 4, we address the issue of implementing the core. For this we develop an exten-

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sive form game for the MPNF problem such that the core of the MPNF game in coalitional function form corresponds to the sub-game perfect Nash equilibria of the extensive form game. In Section 5 we present a result of using the Shapley value as a solution concept for the MPNF game. Finally, in Section 6, we conclude by reiterating our contributions and pointing out future directions of work.

2 The Model

As indicated earlier, the feasible network for forming the multiple unit, single item procurement network may be captured as a directed graph. We call this the procurement feasibility graph $G = (V, E)$, with V as the set of vertices's, two special nodes v_o (origin vertex) and v_t (terminal vertex), and $E \subseteq V \times V$ as the set of edges. With each of the edges $e \in E$ we associate the numbers $c(e)$, $l(e)$, and $u(e)$ to represent the cost, the lower bound, and the upper bound on the capacity of the edge respectively. Now, assume that each of the edges is owned by an agent i where i belongs to a finite set of agents $N = \{1, \ldots, n\}$. We define $\psi : E \to N$ such that $\psi(e) = i$ implies that agent i owns edge e. We let $\mathcal{I}(j)$ and $\mathcal{O}(j)$ represent the set of all incoming and outgoing edges at vertex $j \in V$. Note that we allow a single agent to own multiple edges.

Let $S \subseteq N$ be a coalition of agents. We let E_S represent the set of edges owned by agents in S. We also designate F_S as the flow in the network between the two special nodes v_o and v_t using only the edges E_S that are owned by agents in S. The flow on any edge $e \in E_S$ is designated as $f(e)$. For any flow F_S we denote the set of owners of the edges that facilitate the flow F_S as $\psi(F_S)$. We assume that if multiple units of the item are available to the buyer by using the flow F_S, then it costs $c(F_S)$ and the buyer is willing to compensate the edge owners with a value bF_S where b is the value that the buyer attaches to a single unit of the item. The surplus from such a transaction, is $bF_S - c(F_S)$. The maximum demanded quantity of the buyer is d_{v_t}. The problem now is to:

(a) Maximize the surplus $v_{\mu_b}(S)$ for $S = N$ which is done by solving the surplus maximizing optimization problem given in Equations (1) to (5):

$$v_{\mu_b}(S) \quad = \quad \max \; [bx_{v_t} - \sum_{e \in E_S} c(e) f(e)] \qquad (1)$$

subject to:

$$\sum_{e \in \mathcal{I}(j) \cap E_S} f(e) - \sum_{e \in \mathcal{O}(j) \cap E_S} f(e) = 0, \quad \forall j \in N \backslash \{v_o, v_t\} \qquad (2)$$

$$\sum_{e \in \mathcal{I}(v_t) \cap E_S} f(e) = x_{v_t} \qquad (3)$$

$$\sum_{e \in \mathcal{O}(v_o) \cap E_S} f(e) = x_{v_t} \qquad (4)$$

$$0 \leq x_{v_t} \leq d_{v_t}, \quad \text{and} \quad l(e) \leq f(e) \leq u(e), \forall e \in E_S \qquad (5)$$

(b) Divide the resulting surplus among the agents in the network in a fair way.

These two questions essentially constitute the MPNF problem. We denote this as $\mu_b = (G, N, \psi, b, d_{v_t})$ which in turn induces a cooperative game that can be represented in the characteristic function form as (N, v_{μ_b}) where N is the set of agents and v_{μ_b} is the characteristic function given by solving the optimization problem specified by Equations 1 to 5 for every $S \subseteq N$. We are now interested in finding solutions to this game. That is, we are interested in investigating the non-emptiness of the core of the MPNF game. Formally, it means that we need to allocate the surplus generated to agents such that they obey Equations (6) and (7) where x is an allocation vector, $I(v_{\mu_b})$ is the set of all imputations, and $C(v_{\mu_b})$ the allocations in the core.

$$I(v_{\mu_b}) = \{x \in \Re^n | x_i \geq v_{\mu_b}(\{i\}), \forall i \in N, \sum_{i \in N} x_i = v_{\mu_b}(N)\} \qquad (6)$$

$$C(v_{\mu_b}) = \{x \in I(v_{\mu_b}) | \sum_{i \in S} x_i \geq v_{\mu_b}(S), \forall S \subseteq N\} \qquad (7)$$

The key questions that we need to address regarding the core of the MPNF game are:

1. Does the MPNF game always have a non-empty core?
2. If not, is it at least non-empty under some conditions?

3 Conditions for Non-Emptiness of the Core of MPNF Games

Before examining formally the conditions for the non-emptiness of the core of MPNF games, we first observe that the MPNF game is monotonic and then define some concepts that are useful for the discussion that follows. The approach that we take is similar to the approach in [10]. However, our model is different because we analyse a multi unit procurement problem which maps to a multi-unit flow problem as opposed to the shortest path problem analysed in [10].

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Proposition 1 *The characteristic function of the MPNF game is monotonically non-decreasing, i.e.,* $v_{\mu_b}(S) \le v_{\mu_b}(T), \forall S, T \subseteq N, S \subseteq T$.

Definition 1 *Let* $\mu_b = (G, N, \psi, b, d_{v_t})$ *be a MPNF situation where* G *is a directed acyclic graph whose edges are owned by agents in* N *as indicated by the ownership function* $\psi : E \to N$, b *is the buyer's valuation of a single unit of the item, and* d_{v_t} *is the maximum quantity demanded by the buyer. A flow* F_S *in* G *using only edges owned by agents in* S *is said to be a profitable flow in* μ_b *if* $v_{\mu_b}(S) > 0$.

Definition 2 *Let* μ_b *be an MPNF situation as before. We associate with* μ_b *the MPNF game* (N, v_{μ_b})*. We then say that* μ_b *and* (N, v_{μ_b}) *are non-trivial if* μ_b *has profitable flows or equivalently, if* $v_{\mu_b}(N) > 0$.

Definition 3 *For any MPNF situation* $\mu_b = (G, N, \psi, b, d_{v_t})$ *and its associated game* (N, v_{μ_b})*, an agent* $i \in N$ *is called an f-veto agent (flow veto) agent if he owns at least one edge in every surplus maximizing flow in G.*

3.1 Effect of Ownership Structure on the Non-emptiness of the Core

To formalize the relationship between the ownership structure and the non-emptiness of the core, we proceed by first showing that for any allocation of the surplus to be in the core of the MPNF game, it must be the case that positive allocations of surplus are only made to agents who are a part of the set of f-veto agents. With this result in hand, we then show that the set of f-veto agents must be non-empty and that these agents must own some critical edges in the feasibility graph if the core is to be non-empty.

Lemma 1 *Let* (N, v_{μ_b}) *be a non-trivial procurement network formation game with a set of f-veto agents* V_f*. Let* x *be an imputation of* (N, v_{μ_b}) *such that* $x_i > 0$ *for some agent* $i \in N \setminus V_f$*. Then* $x \notin C(v_{\mu_b})$.

Proof: Since $i \in N \setminus V_f$, $v_{\mu_b}(N) = v_{\mu_b}(N \setminus \{i\})$. Then, taking into account the fact that x is an imputation of (N, v_{μ_b}) and that $x_i > 0$,

$$v_{\mu_b}(N \setminus \{i\}) = v_{\mu_b}(N) = \sum_{j \in N} x_j > \sum_{j \in N \setminus \{i\}} x_j.$$

That is, the surplus value allocated to agents in $N \setminus \{i\}$ is less than the surplus value that these agents can generate by themselves thereby violating the condition of the core. Hence $x \notin C(v_{\mu_b})$. ■

Because of Lemma 1, we understand that the surplus is always divided among the members of the f-veto set V_f and a non f-veto agent never gets a positive share of the surplus. With this we can redefine the MPNF game to reflect the fact that only f-veto agents get a share of the surplus. That is the game (N, v_{μ_b}) can be written as the game $(V_f, \widehat{v}_{\mu_b})$ where the characteristic function $\widehat{v}_{\mu_b}(T)$ is the value that any subset $T \subseteq V_f$ can generate in conjunction with agents who are not in the f-veto set. That is,

$$\widehat{v}_{\mu_b}(T) = v_{\mu_b}(T \cup (N \setminus V_f)) \tag{8}$$

With this definition of a new game involving only the agents in the f-veto set, we now state and prove the main theorem characterizing the effect of the edge ownership structure on the non-emptiness of the core of the MPNF game.

Theorem 1 *Let* (N, v_{μ_b}) *be a non-trivial procurement network formation game with a set of* f*-veto agents* V_f*. Then,* (N, v_{μ_b}) *is balanced if and only if the following two conditions hold: (1)* V_f *is non-empty and* $(V_f, \widehat{v}_{\mu_b})$ *is balanced, (2) Every profitable flow in the procurement network formation scenario* $\mu_b = (G, N, \psi, b, d_{v_t})$ *with which* (N, v_{μ_b}) *is associated contains an edge owned by an* f*-veto agent.*

Proof: We first show the necessity part of the theorem. Suppose that (N, v_{μ_b}) is balanced. We now need to show that conditions (1) and (2) of the theorem hold.

Now, from from the Bondereva-Shapley theorem ([5], [15]), we know that the core of a transferable utility (TU) cooperative game is non-empty if and only if it is balanced. So, because of the assumption of balancedness of the MPNF game (N, v_{μ_b}) which is a TU game, we can say that it has a non-empty core, i.e. $C(v_{\mu_b}) \neq \emptyset$.

Now, consider an imputation x of the surplus value such that it is in the core of the game (N, v_{μ_b}). That is $x \in C(v_{\mu_b})$. Then from Lemma 1, V_f has to be a non-empty set and $x_i = 0$ for all $i \in N \setminus V_f$. Now, denote by $\widehat{x}$ the restriction of x to the set V_f. Clearly, $\widehat{x}$ is an imputation of $(V_f, \widehat{v}_{\mu_b})$. Let $T \subset V_f$. Since $x \in C(v_{\mu_b})$, we have:

$$\sum_{i \in T} \widehat{x}_i = \sum_{i \in T \cup (N \setminus V_f)} x_i \ge v(T \cup (N \setminus V_f)) = \widehat{v}(T).$$

Hence, $\widehat{x} \in C(\widehat{v}_{\mu_b})$, so $(V_f, \widehat{v}_{\mu_b})$ is balanced and condition (1) of the theorem holds.

Now, suppose that there exists a profitable flow F such that $\psi(F) \cap V_f = \emptyset$. Then, $\sum_{i \in N \setminus V_f} x_i \ge v_{\mu_b}(N \setminus V_f) > 0$. This contradicts Lemma 1 and hence there cannot exist a profitable flow where the flow is provided only by agents who are not a part of the set of f-veto agents V_f. Hence, condition (2) of the theorem holds.

This proves the necessity part of the theorem. □

We now show the sufficiency part of the theorem. Suppose that the MPNF game (N, v_{μ_b}) satisfies conditions 1 and 2.

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We now need to show that the core of this game is non-empty.

Consider an imputation $\widehat{x} \in C(\widehat{v}_{\mu_b})$. We now define $x \in \Re^n$ in the following way: $x_i = \widehat{x}_i$ if $i \in V_f$, else $x_i = 0$ for all $i \in N \setminus V_f$. It should be noted that the x we have constructed is now an imputation of (N, v_{μ_b}). Also we know that the game (N, v_{μ_b}) is monotonic from Proposition 1. Now consider any coalition of agents $S \subseteq N$. There are two cases to consider here: either (a) $S \bigcap V_f \neq \emptyset$ or (b) $S \bigcap V_f = \emptyset$.

Case (a): For every $S \subset N$ with $S \cap V_f \neq \emptyset$, since $\widehat{x} \in C(\widehat{v}_{\mu_b})$, we have:

$$\sum_{i \in S} x_i = \sum_{i \in S \cap V_f} \widehat{x}_i \geq \widehat{v}_{\mu_b}(S \cap V_f) \geq v_{\mu_b}(S).$$

The above equation is nothing but the core condition for all coalitions $S \subseteq N$ where $S \bigcap V_f \neq \emptyset$.

Case (b): Now consider all coalitions $S \subseteq N$ where $S \bigcap V_f = \emptyset$. Since (N, v_{μ_b}) satisfies condition (2) in the statement of theorem, we can infer that for all coalitions S with $S \bigcap V_f = \emptyset$ we must have $v_{\mu_b}(S) = 0$. It is easy to see that any allocation of the surplus of the MPNF game will always satisfy the core condition for all those coalitions S where $S \bigcap V_f = \emptyset$.

With this we have shown that for all coalitions $S \subseteq N$, we have $\sum_{i \in S} x_i \geq v_{\mu_b}(S), \forall S \subseteq N$. This is nothing but the condition for the non-emptiness of the core of the MPNF game (N, v_{μ_b}). So, sufficiency is proved. □

So the theorem holds. ■

Managerial Implications of Theorem 1: Firstly, from a procurement network design point of view, given the MPNF scenario $\mu_b = (G, N, \psi, b, d_{v_t})$, the agents who have the maximum bargaining power in the feasibility graph can be marked out. Also, notice that the statement of the theorem says that the non-emptiness of the set of f-veto agents is the criterion for the core to be non-empty. Now assume that the buyer was himself included in the feasibility graph as an agent who owns a dummy edge from the terminal node v_t to another dummy node, say v_d and the feasibility graph now includes both the dummy edge and the node. It is easy to see that in this case the set of f-veto agents is always non-empty and minimally includes the buying agent. By carrying out an analysis on this new graph, the buyer can see with which other suppliers in the network he will have to share the bargaining power. Such an analysis, often the subject of *supplier base/network optimization* within large global supply chains, can help the buyer to map out a strategy for supplier development so that the bargaining power of agents can be limited while simultaneously ensuring the stability of the network.

Secondly, from a diagnostic point of view, it provides support to the suppliers also. Recall that the result tells us that only f-veto agents get a positive share of the surplus. So, suppliers need to ensure that they are a part of the f-veto set if they hope to garner a share of the surplus. An analysis of the network will tell them how much they need to improve their cost competitiveness in order to be in the f-veto set. Also, it provides them with insight into what complementary capabilities or additional capabilities they need to acquire by buying out edges in the network that would make them a f-veto agent.

3.2 Effect of Demanded Quantity on the Non-emptiness of the Set of f-veto Agents

In this section, we focus on formalizing the relationship between the demanded quantity and the non-emptiness of the core for a given procurement feasibility graph. From Lemma 1 and Theorem 1 we understand that the existence of a non-empty set of f-veto agents is crucial to the non-emptiness of the core of the MPNF game. We now formally show the relationship between the demanded quantity and the non-emptiness of the core through its relationship with the non-emptiness of the set of f-veto agents.

Theorem 2 *Let $\mu_b = (G, N, \psi, b, d_{v_t})$ be a* MPNF *scenario. Then there exists $D \in (0, \infty)$ such that the set of f-veto agents is non-empty for all $d_{v_t} \geq D$.*

Proof: Recall that the set of f-veto agents V_f are those agents that own an edge in every surplus maximizing flow in the network. Because of this we can infer that there are at least two coalitions of agents $S_1, S_2 \subseteq N, S_1 \bigcap S_2 = \emptyset$ who can provide a surplus maximizing flow for a given procurement scenario $\mu_b = (G, N, \psi, b, d_{v_t})$ when the set V_f is empty. That is we have:

$$v_{\mu_b}(N) = v_{\mu_b}(S_1) = v_{\mu_b}(S_2). \tag{9}$$

It is clear that the limiting constraint in this surplus maximization problem is the maximum demanded quantity d_{v_t}. Set $d_{v_t} \Leftarrow 2 * d_{v_t}$. Now, $S_1 \cup S_2$ is a surplus maximizing coalition for the procurement scenario $\mu_b = (G, N, \psi, b, 2 * d_{v_t})$. Now, either $S_1 \cup S_2$ yields a f-veto set of agents in which case we are done or we repeat the procedure until such a set is found. And this is guaranteed since there are only a finite set of agents. So, by this procedure we are guaranteed to find a number $D \in [0, \infty)$ such that $d_{v_t} = D$ induces a set of f-veto agents. ■

Managerial Implications of Theorem 2: Recall from Theorem 1 that one of the conditions for the core to be non-empty is that the set of f-veto agents should be non-empty. Theorem 2 is essentially a link to the non-emptiness of the core through the set of f-veto agents. From a managerial perspective, the theorem suggests that if the buyer

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desires the formation of a stable procurement network, then the maximum demand that he specifies will have to be carefully chosen if the set of f-veto agents is to be non-empty.

3.3 Effect of Buyer's Valuation on the Non-emptiness of the Core

In this section we focus on formalizing the relationship between the buyer's valuation for each unit of the item and the non-emptiness of the core of the MPNF game for special case when the maximum demanded quantity if unity.

To do this we proceed by first showing through Lemma 2 that if the special case of the MPNF scenario $\mu_b = (G, N, \psi, b, d_{v_t} = 1)$ induces a game (N, v_{μ_b}) whose core is non-empty then for every other MPNF scenario where the buyer's valuation $\widehat{b}$ of the item is lower, the induced game $(N, v_{\mu_{\widehat{b}}})$ continues to have a non-empty core. We then show through Theorem 3 that either the core of the MPNF game is always non-empty or there exists a threshold value of the buyer's valuation below which the core is always non-empty.

Lemma 2 *Let $\mu_b = (G, N, \psi, b, d_{v_t} = 1)$ be a procurement network formation scenario and the associated cooperative game be (N, v_{μ_b}). If (N, v_{μ_b}) is balanced, then $(N, v_{\mu_{\widehat{b}}})$ is balanced for all $\widehat{b} \in [0, b]$ where $\mu_{\widehat{b}} = (G, N, \psi, \widehat{b}, d_{v_t} = 1)$.*

Proof: Let $x = (x_1, x_2, \ldots, x_n)$ be an imputation of (N, v_{μ_b}). Then x is an element of the core of the game if and only if $\sum_{i \in S} x_i \geq v_{\mu_b}(S)$ for every $S \subseteq N$ that has a profitable flow in μ_b. Now, consider some $x = (x_1, x_2, \ldots, x_n) \in C((N, \mu_b))$ and any $\widehat{b} \leq b$. The corresponding procurement network formation scenario $\mu_{\widehat{b}}$ induces the cooperative game $(N, v_{\mu_{\widehat{b}}})$. There are two cases to consider for the newly induced game. Either (a)$v_{\mu_{\widehat{b}}}(N) = 0$ or (b) $v_{\mu_{\widehat{b}}}(N) > 0$

Case (a): When $v_{\mu_{\widehat{b}}}(N) = 0$.

If $v_{\mu_{\widehat{b}}}(N) = 0$, then because the MPNF game is monotonic, it is clearly the case that there is no subset of agents who can create a positive surplus. That is $v_{\mu_{\widehat{b}}}(S) = 0, \forall S \subseteq N$. Then for any balanced vector $\theta = (\theta(S))_{S \in L(N)}$, where $L(N) = \{S | S \subseteq N, S \neq \emptyset\}$, it is clear that $\sum_{S \subseteq N} \theta(S)\ v_{\mu_{\widehat{b}}}(S) = 0 = v_{\mu_{\widehat{b}}}(N) = 0$. This is nothing but the condition for balancedness of the game $(N, v_{\mu_{\widehat{b}}})$ (refer the Definition **??**). Hence the lemma holds in this case.

Case (b): When $v_{\mu_{\widehat{b}}}(N) > 0$.

For this note that the difference in surplus that the grand coalition makes when the buyer's valuation is b and $\widehat{b}$ is exactly equal to the difference in the valuation itself. This is given by the following equation:

$$v_{\mu_b}(N) - v_{\mu_{\widehat{b}}}(N) = b - \widehat{b} \geq 0 \quad (10)$$

We have already picked $x = (x_i)_{i \in N}$ as an imputation of the surplus that is in the core of the game (N, v_{μ_b}). Now we choose an imputation $\widehat{x}$ of $(N, v_{\mu_{\widehat{b}}})$ such that the following condition holds:

$$x_i - \widehat{x}_i \geq 0, \forall i \in N. \quad (11)$$

From equations 10 and 11 we can deduce the following relation:

$$\sum_{i \in S} x_i - \sum_{i \in S} \widehat{x}_i \leq b - \widehat{b} \quad (12)$$

$$\Rightarrow \textstyle\sum_{i \in S} \widehat{x}_i \geq b - c(F_S) - b + \widehat{b}$$
$$\Rightarrow \textstyle\sum_{i \in S} \widehat{x}_i \geq v_{\mu_{\widehat{b}}}(S)$$

Starting from an imputation which is in the core of the original game we have shown that it is possible to construct an imputation which is in the core of the new game whose budget is bounded by the budget of the original game for which the core is non-empty. And by the Bondereva-Shapley theorem we know that the core of a cooperative game is nonempty if and only if it is balanced. Since the new game has a non-empty core we can infer that it is also balanced and the lemma holds for this case also. So, the Lemma holds. ■

We now state the main theorem characterizing the relationship between the buyer's valuation and the non-emptiness of the core of the MPNF game when the demanded quantity is unity. The proof is a simple extension of the argument that appears in [10] and hence is omitted to economize on space.

Theorem 3 *Let $\mu_b = (G, N, \psi, b, d_{v_t} = 1)$ be a procurement network formation scenario. Then either the cooperative game (N, v_{μ_b}) associated with the scenario is balanced for all $b \in [0, \infty)$ or there exists $B \in [0, \infty)$ such that (N, v_{μ_b}) is balanced if and only if $b \leq B$.*

Managerial Implications of Theorem 3: From a managerial perspective, Theorem 3 suggests that if the buyer desires the formation of a stable procurement network in the sense that re-contracting is precluded, then the budget that he announces will have to be carefully chosen. A very low budget may mean that there is no profitable flow in the network and hence a transaction does not occur and a high budget will tend to engage the suppliers in protracted negotiations which do not seem to terminate in any agreement precisely because the core is empty.

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4 An Extensive Form Game to Implement the Core

In general the core as a solution concept only points out that an allocation in the core is immune to re-contracting either by the grand coalition of all agents or sub-coalitions of agents. It takes an exogenous view of the cooperative scenario and points out that agents when given *sufficient* time and message space for negotiations are likely to converge to one of the allocations in the core that are themselves indicated axiomatically. From an implementation viewpoint however, this can be cumbersome. That is, in actual practice, in a MPNF scenario, we do not normally have the advantage of a central agency or a social planner who can point out to the agents the strategies they must adopt to obtain payoffs that are in the core of the game. We must have a way to allow agents to non-cooperatively achieve the desired outcome.

As we know the core can essentially be viewed as a social choice correspondence from the space of characteristic function values to the space of allocations. That is, the core C of a cooperative game (N, v) is a social choice correspondence given by: $C : \Re^{|L(N)|} \rightarrow \Re^N$ where $L(N) = \{S | S \subseteq N,\ S \neq \emptyset\}$

Implementation theory provides us with a body of ideas to implement such social choice correspondences. We point the reader to [13] for a comprehensive survey of this field. The idea here is to construct detailed rules of bargaining in an extensive form game and show that the set of non-cooperative equilibria of this game, possibly Nash equilibria or some refinement thereof, coincides with the outcomes of the social choice correspondence.

We have constructed an extensive form game Γ such that for every scenario in the class of MPNF games, the sub-game perfect Nash equilibrium outcomes of Γ coincide with the core allocations of the MPNF game μ_b. The extensive form game or mechanism that we construct is essentially a two stage game with agents moving simultaneously in Stage 0 and moving sequentially in Stage 1. Our aim for Stage 0 is two fold: First, we would like to have a status-quo allocation that agents will revert to in case of disagreement in Stage 1; secondly we seek an ordering of agents so that they may move sequentially in Stage 1. In Stage 1, we allow agents to make sequential moves, such as proposing, accepting, or rejecting offers. These moves are based on an ordering of agents computed at the end of Stage 0. The game ends after each of the agents has made at most one move in Stage 1. The leaf nodes of the game tree indicate whether an agreement has been reached and if so a split of the surplus. For details of the extensive form game Γ we point the reader to [7]. The key result is summarized by the theorem below.

Theorem 4 *The extensive form mechanism Γ implements in sub-game perfect Nash equilibrium the core of the class of* **MPNF** *games.*

5 The Shapley Value of the Procurement Network Formation Game

In this section, we consider the multiple unit single item procurement network formation problem as a surplus maximizing network flow cooperative game, where the buyer is included as a game theoretic agent. We investigate the implications on ownership structure of using the Shapley value as a solution concept for this game.

In fact, the Shapley value seems to make positive allocations of the surplus to agents who own edges in any flow that generate a positive surplus but are not really a part of the surplus maximizing flow. This can be disconcerting in a procurement situation where the surplus allocation rule gives away money to agents who do not actually participate in the surplus maximizing flow. This weakness of the Shapley value is precisely because of the following reasons:

First, it assumes that the grand coalition of all agents will be formed. Secondly, it is derived entirely from a specification of the characteristic function of the game and not from what is *behind* the characteristic function. That is, it relies completely on the strategic structure of the game itself rather than the bargaining positions of the players in the process of coalition formation.

Guided by this observation, it is important for us to gain an understanding of the scenarios where the Shapley Value makes allocations of surplus value to the buyer and only those agents who own edges in the surplus maximizing flow.

Definition 4 *The set of all agents, denoted $S_M(F)$, who own edges in a surplus maximizing flow F of the procurement graph are called the SM-Agents, i.e., $S_M = \{i = \psi(e), i \in N : e \in \psi(F)\}$ associated with the surplus maximizing flow F.*

It can easily be shown that the MPNF game with the addition of the buyer as a game theoretic agent is zero-monotonic as in the proposition below. The main theorem of this section whose proof is available in [7] follows immediately thereafter.

Proposition 2 *The characteristic function of the MPNF game when the buyer is included as an agent is zero-monotonic, i.e., $v_{\mu_b}(N) \geq v_{\mu_b}(N \backslash \{i\}) + v_{\mu_b}(\{i\}), \forall i \in N$.*

Theorem 5 *If the Shapley value rule allocates all the surplus value in the MPNF game only to agents $i \in S_M$ then for every flow F_S provided by a coalition S that includes an agent $i \notin S_M$, either*

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1. *F_S is not profitable, i.e, $v_{\mu_b}(S) = 0$ or*

2. *if F_S is profitable, then we have $\psi(F_S) \cap S_M \neq \emptyset$, and there is a set $S_{S_M} \subset \psi(F_S) \cap S_M$ such that $v(S_{S_M}) = v(\psi(F_S))$.*

6 Conclusions and Future Work

In this paper we have mainly considered the multi unit single item procurement scenario where both the cost and valuation information is completely known to the buyer and the suppliers. We have considered these scenarios in the context of suppliers sharing very open relationships. However, in many procurement contexts such transparency in information may not be forthcoming. In these cases it is useful to consider incomplete information versions of the cooperative model considered in this paper. We offer below some threads of investigation which would be useful in furthering our understanding of the procurement network formation problem.

1. Extend the model to analyze (a) the multi item multi unit PNF problem and (b) applicability of other solution concepts viz, nucleolus, kernel, and the ϵ-core.

2. Extend the analysis to model scenarios of incomplete information. This would require the appropriate extensions in the solution concepts such as the incentive compatible core. See [7] for a review of these concepts and the analysis in the context of the PNF problem.

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