Trade Determination in Multi-Attribute Exchanges

S Kameshwaran * Y Narahari
Electronic Enterprises Laboratory
Department of Computer Science and Automation
Indian Institute of Science
Bangalore - 560 012, India
{kameshn, hari}@csa.iisc.ernet.in

Abstract

Electronic exchanges are double-sided marketplaces that allow multiple buyers to trade with multiple sellers, with aggregation of demand and supply across the bids to maximize the revenue in the market. Two important issues in the design of exchanges are (1) trade determination (determining the number of goods traded between any buyer-seller pair) and (2) pricing. In this paper we address the trade determination issue for one-shot, multi-attribute exchanges that trade multiple units of the same good. The bids are configurable with separable additive price functions over the attributes and each function is continuous and piecewise linear. We model trade determination as mixed integer programming problems for different possible bid structures and show that even in two-attribute exchanges, trade determination is $\mathcal{NP}$-hard for certain bid structures. We also make some observations on the pricing issues that are closely related to the mixed integer formulations.

1 Introduction

Markets play a central role in any economy and facilitate exchange of information, goods, services, and payments. They are intended to create value for buyers, sellers, and society at large. Markets have three main functions [2]: (1) matching buyers to sellers; (2) facilitating exchange of information, goods, services, and payments associated with a market transaction; and (3) providing an institutional infrastructure. Internet-based markets leverage information technology to perform these functions with increased effectiveness and reduced transaction costs, leading to more efficient, friction-free markets. Exchanges are double-sided marketplaces where both buyers and sellers submit bids for trading. We refer to sellers' bids as *offers* whenever it is required to differentiate them from that of buyers. The exchanges differ in functionality with respect to timing of clearing, number of bid submissions, pricing, aggregation, and the varieties of goods traded. *Iterative* exchanges iterate the bid submission, clearing, and information disclosure to the bidders. The clearing of the exchange is *continuous* if trading is triggered by the arrival of bids and *periodic* if trading occurs after a prespecified interval of time. In *one-shot* exchanges, agents submit sealed bids once during the specified bidding interval and the market is cleared after the termination of the bidding time. Based on the pricing scheme, the exchanges can be *uniformly-priced* or *discriminately-priced*. In uniformly-priced exchanges, agents pay the same price for the same good whereas in discriminatory-pricing exchanges agents are differentially priced. *Homogeneous* exchanges trade multiple units of a single good, *combinatorial* exchanges allow bids for bundles of different goods, and *multi-attribute* exchanges allow bids that specify different attributes in addition to quantity and price for goods. Based on the aggregation allowed among the bids, the exchanges can be *sell-side*, *buy-side*, or *completely* aggregated [15]. The *continuous double auction* markets for stock trading are continuous, discriminately-priced, homogeneous exchanges whereas the *call markets* used for the daily opening on the New York, American, and Tokyo Stock Exchanges [12] are one-shot, uniformly-priced, homogeneous, completely aggregated exchanges.

The two core problems in the design of an exchange are *allocation* and *pricing* [15]. We call the allocation problem as the *trade determination problem* (TDP), which determines the quantity of goods traded between every pair of buyers and sellers. In this paper, our interest is in the TDP for one-shot, multi-attribute, homogeneous exchanges with configurable bids. The functional form of configurable bids considered in this paper corresponds to separable additive functions over the attributes with each function continuous and piecewise linear. We architect a set of exchanges

*This research is supported in part by IBM Research Fellowship awarded to the first author by IBM India Research Lab, New Delhi

for different possible functional forms and bid structures, and model their TDPs as mixed integer programming (MIP) problems. We also show that trade determination is $\mathcal{NP}$-hard for certain bid structures even in a two-attribute exchange.

The rest of the paper is organized as follows. Related literature is discussed in Section 2. In Section 3, we propose a price-quantity two-attribute exchange, where the price is given as a continuous, piecewise linear function of the quantity. MIPs for varying price functions and bid patterns are presented. Section 3 extends the models for multiple attributes. Though the focus of the paper is on the TDP, we highlight certain pricing issues in Section 4 as they are closely related with the TDP. Conclusions and future research are noted in Section 5.

2 Related Literature

Multi-dimensional markets have become popular with the advent of Internet based marketplaces [3], with applications varying from procurement in private marketplaces [1] to tourism [4]. Much of the literature is on multi-attribute auctions, which are single sided market mechanisms with a single auctioneer and multiple bidders. The auctioneer usually has a value or utility function for each attribute and weights that capture the relative importance of the attributes. Using a scoring function, the auctioneer quantifies each bid and selects the best that satisfies his requirements. The scoring function generally used is the additive scoring function derived from multi-attribute utility theory [10]. The bidding language used in the above auctions are (*attribute, value*) pairs. To further automate negotiations, configurable bids were used in [5]. With configurable bids, the auctioneer can configure the product or service offered by the bidder. Configurable bids express price as a function of the attributes, which enables the auctioneer to choose the best configuration that suits his price.

Multi-attribute exchanges (MAX) are a class of double-sided market mechanisms with both buyers and sellers posting multi-attribute bids. The scoring function technique with (*attribute, value*) bidding language is not a feasible extension to MAX. This is because deriving a single scoring function that is acceptable for both buyers and sellers is not possible since their interests are generally conflicting. The more suitable bidding language is that of configurable bids. A MAX mechanism with configurable bids where the price is given as a function of quantity and lead time was proposed in [8]. Computational aspects of clearing homogeneous exchanges were discussed in [7] and those of combinatorial exchanges were discussed in [17, 16, 11]. In this paper we focus on MAX mechanisms with configurable bids that are separable and additive functions of attributes, where each function is a continuous piecewise linear function.

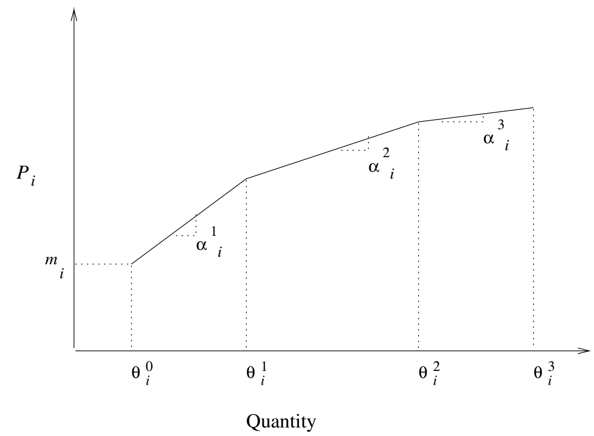

Figure 1. Piecewise linear concave P_i

3 Two-attribute Exchanges

The simplest case of a multi-attribute exchange is a two-attribute exchange (TAX) with price and quantity as attributes. A configurable bid for this exchange would express price as a function of quantity. The bid submitted by buyer i is a quantity-price pair $([\underline{b}_i, \overline{b}_i], P_i)$, which means that the buyer is willing to trade any quantity in the range $[\underline{b}_i, \overline{b}_i]$ at the price given by the function P_i. The price function P_i, we consider, is a continuous piecewise linear function on $[\underline{b}_i, \overline{b}_i]$ as shown in Figure 1. The price is the total price (not the unit price) at which the buyer is willing to trade as a function of quantity. The P_i shown in Figure 1 can be compactly represented by tuples of break points and slopes $((\theta_i^0, \ldots, \theta_i^{k(i)}), (\alpha_i^1, \ldots, \alpha_i^{k(i)}), m_i)$ where $k(i)$ is the number of linear segments and m_i is the price at $\theta_i^0 = \underline{b}_i$. The break points $\theta_i^0 = \underline{b}_i < \theta_i^1 \ldots < \theta_i^{k(i)} = \overline{b}_i$ represent the points at which the function changes slope and the corresponding slopes are decreasing ($\alpha_i^1 > \alpha_i^2 > \ldots > \alpha_i^{k(i)}$) to reflect negotiation of price over quantity. The offers submitted by sellers $([\underline{a}_j, \overline{a}_j], Q_j)$ with Q_j as a price function with $l(j)$ linear segments, represented by $((\delta_j^0, \ldots, \delta_j^{l(j)}), (\beta_j^1, \ldots, \beta_j^{l(j)}), n_j)$ have the same interpretation ($\delta_j^0 = \underline{a}_j < \delta_j^1 \ldots < \delta_j^{l(j)} = \overline{a}_j$, $\beta_j^1 > \beta_j^2 > \ldots > \beta_j^{l(j)}$ and n_j is the price at $\underline{a}_j$). Note that both P_i and Q_j are increasing concave functions. P_i denotes the negotiating behavior of buyer i where he is willing to buy more goods if the price is reduced and Q_j denotes the volume discount strategy of seller j, where he is willing to reduce the price if more goods are bought. In the following subsections we progressively develop four models of TAX from the simplest case to more general ones and provide MIP formulations for their respective TDPs.

Proceedings of the IEEE International Conference on E-Commerce (CEC'03)
0-7695-1969-5/03 $17.00 © 2003 IEEE

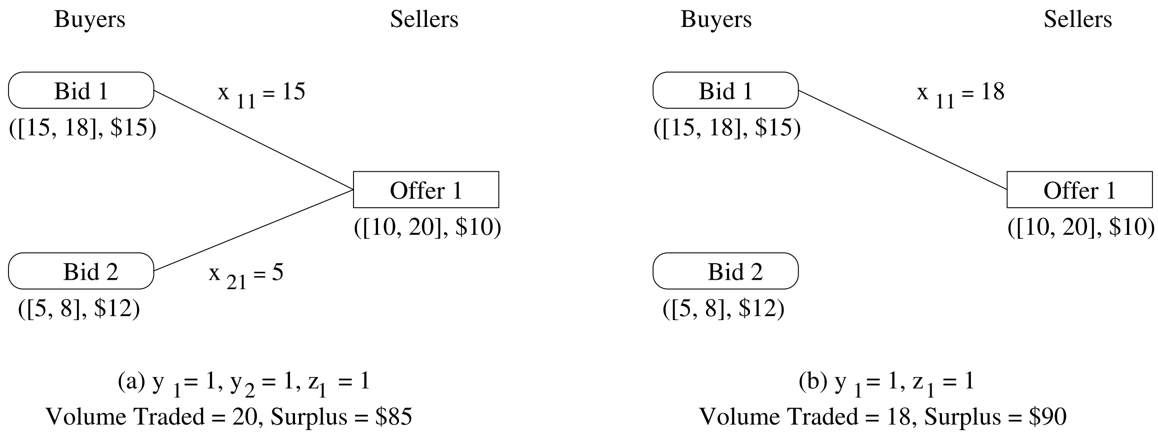

Figure 2. More surplus with less number of bids

3.1 TAX-1 ($\underline{b}_i = 0, \underline{a}_j = 0, k(i) = l, l(j) = 1, \ \forall i, j$)

The first model is the simplest of all with no lower bounds on the quantity required and the price functions are linear functions with slope α_i for P_i and β_j for Q_j. Following is the integer programming (IP) formulation for TDP with the objective of maximizing the surplus in the market.

$$\max \sum_i (\alpha_i \sum_{j \in B_i} x_{ij}) - \sum_j (\beta_j \sum_{i \in S_j} x_{ij}) \quad (1)$$

subject to

$$\begin{aligned} \sum_{j \in B_i} x_{ij} &\leq \overline{b}_i \quad \forall i \\ \sum_{i \in S_j} x_{ij} &\leq \overline{a}_j \quad \forall j \\ x_{ij} &\in \mathcal{Z}^+ \quad \forall i, \ j \in B_i \end{aligned}$$

The objective function maximizes the surplus generated out of the trade. The decision variable x_{ij} denotes the number of goods traded between buyer i and seller j. The set B_i (S_j) denotes the set of compatible offers (bids) for bid i (offer j). This set defines which bid can trade with which offer. For the current model, these sets hardly play any role and their purpose will become clear in the subsequent models. One can easily see that the objective function is equivalent to $\sum_{i,j} (\alpha_i - \beta_j) x_{ij}$ and the problem is a variant of the transportation problem, which can be solved in polynomial time using transportation algorithms [13]. The integrality constraint $x_{ij} \in \mathcal{Z}^+$ can be relaxed to $x_{ij} \geq 0$, as transportation algorithms always find integral x_{ij} whenever $\overline{a}_j$ and $\overline{b}_i$ are integers [13]. The objective function can be altered to maximize the trade volume $\sum_{i,j} x_{ij}$ with the extra constraint $\sum_i (\alpha_i \sum_{j \in B_i} x_{ij}) \geq \sum_j (\beta_j \sum_{i \in S_j} x_{ij})$ so that the market does not run in deficit. This constraint can be removed if we define $B_i = \{j : \alpha_i - \beta_j \geq 0\}$ and $S_j = \{i : \alpha_i - \beta_j \geq 0\}$ and can be solved using transportation algorithms.

3.2 TAX-2 ($k(i) = 1, l(j) = 1, \ \forall i, j$)

In TAX-2, the agents have non-zero lower bounds on the quantity, i.e. the buyer (seller) is willing to trade any quantity in range $[\underline{b}_i, \overline{b}_i]$ ($[\underline{a}_j, \overline{a}_j]$) and nothing outside this range. This requires additional 0-1 decision variables to accept/reject a bid for trading. The MIP formulation for the TDP is as follows:

$$\max \sum_i (y_i m_i + \alpha_i \sum_{j \in B_i} x_{ij}) - \sum_j (z_j n_j + \beta_j \sum_{i \in S_j} x_{ij}) \quad (2)$$

subject to

$$\begin{aligned} y_i \underline{b}_i \leq \ & \textstyle\sum_{j \in B_i} x_{ij} \leq \ y_i \overline{b}_i \quad \forall i && (3) \\ z_j \underline{a}_j \leq \ & \textstyle\sum_{i \in S_j} x_{ij} \leq \ z_j \overline{a}_j \quad \forall j && (4) \\ & x_{ij} \geq 0 \quad \forall i, j \in B_i && (5) \\ & y_i \in \{0, 1\} \quad \forall i && (6) \\ & z_j \in \{0, 1\} \quad \forall j && (7) \end{aligned}$$

The y_i and z_j are 0-1 variables that reject/select a bid or an offer. When the 0-1 variables are set, the resulting problem is an interval transportation problem [13] that can be solved using transportation algorithms. Due to this special transportation structure, we have not constrained x_{ij} to be integers. The binary variables y_i and z_j are required for two reasons. One, accepting all bids and offers may not maximize the value of (2) as shown in Figure 2. The attributes of the bids in Figure 2 are *quantity range* and *unit price*: for example, *Bid 1* ([15, 18], $15) means that the buyer is willing to buy any number of goods in the range [15, 18] at unit price $15 and no goods outside this range. Figure

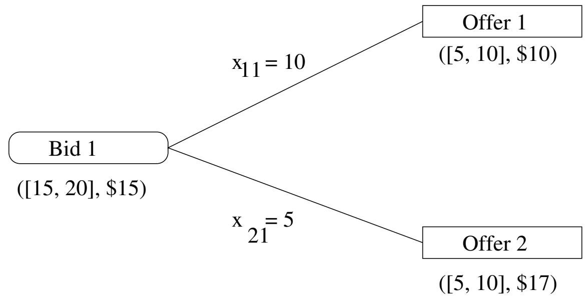

Figure 3. Trade with negative surplus

2(a) shows the case when both bids are selected: total volume traded is $x_{11} + x_{21} = 20$ and the surplus generated is $15 \times (\$15 - \$10) + 5 \times (\$12 - \$10) = \$85$. When only *Bid 1* is selected (Figure 2(b)), total volume traded is $x_{11} = 18$ and the surplus generated is $18 \times (\$15 - \$10) = \$90$. Thus more surplus can be generated by selecting less number of bids and by trading lower volumes. This phenomenon is a variant of the *more-for-less* paradox [6] of transportation problems, and more generally of linear programming problems. The paradox says that it is possible to send more flow from supply to demand nodes at lower cost, even if all arc costs are positive. The second reason for the binary decision variables is that the problem may be infeasible after accepting all bids and offers. For the above problem to be feasible, following inequalities need to be satisfied (assuming that any bid can trade with any offer and vice versa):

$$\sum_i \overline{b}_i \geq \sum_j \underline{a}_j$$
$$\sum_j \overline{a}_j \geq \sum_i \underline{b}_i$$

Hence, 0-1 variables are required to judiciously select bids and offers such that the problem is feasible and (2) is maximized. This model is close to the MAX of [8], where Benders' Decomposition based solution technique is proposed for solving the TDP. It is worth noting here that trades with only positive surplus ($\alpha_i - \beta_j > 0$) occur in TAX-1; TAX-2 can have trades with negative surplus (even though (2) can be positive) as shown in Figure 3. *Bid 1* trades five units with *Offer 2* at a negative surplus of -\$2 per unit but the total surplus generated in the exchange is positive ($10 \times (\$15 - \$10) + 5 \times (\$15 - \$17) = \$40$).

Now we show that the decision version of TAX-2 (DTAX-2) is $\mathcal{NP}$-complete by reducing from capacitated facility location problem (CFLP) [14] (thus TAX-2 is $\mathcal{NP}$-hard).

Definition 1 (DTAX-2) *We are given a set of bids* $B = \{([\underline{b}_i, \overline{b}_i], \alpha_i, m_i)\}$, *a set of offers* $S = \{([\underline{a}_j, \overline{a}_j], \beta_j, n_j)\}$, *and a goal* G. *We are asked whether there exist* $B' \subseteq B$, $S' \subseteq S$ *and assignment* $x_{ij} \geq 0$, *such that*

$$\underline{b}_i \leq \sum_{j \in S'} x_{ij} \leq \overline{b}_i \quad \forall i \in B' \tag{8}$$
$$\underline{a}_j \leq \sum_{i \in B'} x_{ij} \leq \overline{a}_j \quad \forall j \in S' \tag{9}$$

and $-\sum_{i \in B'} m_i - \sum_{i \in B'} \sum_{j \in S'} (\alpha_i - \beta_j) x_{ij} + \sum_{j \in S'} n_j < G$.

Definition 2 (CFLP) *We are given a set of facilities* $\tilde{S}$, *a set of clients* $\tilde{B}$ *and a goal* $\tilde{G}$. *Each facility* $j \in \tilde{S}$ *incurs a fixed cost* $\tilde{n}_j$ *when opened and has a maximum capacity* $\tilde{a}_j$. *Each client* $i \in \tilde{B}$ *has a demand* $\tilde{b}_i$. *The unit cost of serving from capacity* j *to client* i *is* $\tilde{c}_{ij}$. *We are asked whether there exists a set* $\tilde{S}' \subseteq \tilde{S}$ *of facilities that can be opened and each* $j \in \tilde{S}'$ *serving* $\tilde{x}_{ij} (\geq 0)$ *units to client* i, *such that*

$$\sum_{j \in \tilde{S}'} \tilde{x}_{ij} = \tilde{b}_i \quad \forall i \in \tilde{B} \tag{10}$$
$$\sum_{i \in \tilde{B}'} \tilde{x}_{ij} \leq \tilde{a}_j \quad \forall j \in \tilde{S}' \tag{11}$$

and $\sum_{j \in \tilde{S}'} \tilde{n}_j + \sum_{i \in \tilde{B}} \sum_{j \in \tilde{S}'} \tilde{c}_{ij} \tilde{x}_{ij} < \tilde{G}$.

Theorem 1 *DTAX-2 is* $\mathcal{NP}$*-complete.*

Proof: To show that DTAX-2 is in $\mathcal{NP}$, we observe that specifying a solution is to choose subsets B' and S', and assign non-negative values to $x_{ij} \ \forall i \in B', \ j \in S'$. Given such a solution, we can verify whether it meets our requirements in polynomial time. To show $\mathcal{NP}$- hardness, we reduce an arbitrary CFLP instance to the following DTAX-2 instance:

- $B = \tilde{B}, \ S = \tilde{S}, \ n_j = \tilde{n}_j, m_i = M (>> 0$, arbitrarily large number);
- $\underline{a}_j = 0, \ \overline{a}_j = \tilde{a}_j, \ \underline{b}_i = \overline{b}_i = \tilde{b}_i$, and $(\alpha_i - \beta_j) \equiv -\tilde{c}_{ij}$;
- Set goal $G = \tilde{G} - \sum_{i \in B} m_i$.

It is obvious that the above reduction can be done in polynomial time. Let us show now that the reduction is valid. Suppose there is a solution to CFLP instance, i.e. there exists $\tilde{S}' \subseteq \tilde{S}$ and $\tilde{x}_{ij}$ such that (10) and (11) are satisfied and $\sum_{j \in \tilde{S}'} \tilde{n}_j + \sum_{i \in \tilde{B}} \sum_{j \in \tilde{S}'} \tilde{c}_{ij} \tilde{x}_{ij} < \tilde{G}$. Choose $S' = \tilde{S}'$, $x_{ij} = \tilde{x}_{ij}$ and $B' = \tilde{B}$. This solution satisfies (8) and (9), and satisfies the goal G:

$$\sum_{i \in \tilde{B}} \sum_{j \in \tilde{S}'} \tilde{c}_{ij} \tilde{x}_{ij} + \sum_{j \in \tilde{S}'} \tilde{n}_j \quad < \quad \tilde{G}$$

$$\Rightarrow \quad -\sum_{i\in B'} m_i - \sum_{i\in B'}\sum_{j\in S'}(\alpha_i-\beta_j)x_{ij} + \sum_{j\in S'} n_j$$
$$< -\sum_{i\in B} m_i + \tilde{G} = G$$

So we have a solution to DTAX-2 instance. Now suppose DTAX-2 instance has a solution, i.e. there exists $S' \subseteq S$, $B' \subseteq B$, and $x_{ij} \geq 0$ satisfying (8) and (9), such that $-\sum_{i\in B'} m_i - \sum_{i\in B'}\sum_{j\in S'}(\alpha_i-\beta_j)x_{ij} + \sum_{j\in S'} n_j < G$. First we note that $B' = B$. Suppose $B' \subset B$, then $-\sum_{i\in B'} m_i - \sum_{i\in B'}\sum_{j\in S'}(\alpha_i-\beta_j)x_{ij} + \sum_{j\in S'} n_j < \tilde{G} - \sum_{i\in B} m_i$ cannot be true as we have assumed $m_i = M$ to be an arbitrarily large number. Now choose $\tilde{S}' = S'$, $\tilde{B}' = B$, and $\tilde{x}_{ij} = x_{ij}$. This solution satisfies (10) and (11), and satisfies the goal $\tilde{G}$:

$$-\sum_{i\in B'} m_i - \sum_{i\in B'}\sum_{j\in S'}(\alpha_i-\beta_j)x_{ij} + \sum_{j\in S'} n_j$$
$$< \tilde{G} - \sum_{i\in B} m_i$$
$$\Rightarrow \quad \sum_{i\in \tilde{B}}\sum_{j\in \tilde{S}'} \tilde{c}_{ij}\tilde{x}_{ij} + \sum_{j\in \tilde{S}'} \tilde{n}_j \quad < \quad \tilde{G}$$

So there is a solution to CFLP instance. ■

3.3 TAX-3 ($\underline{b}_i = 0, \underline{a}_j = 0, \forall i, j$)

In TAX-3, the agents have piecewise linear, concave price functions with zero lower bounds on quantity. The objective function is maximization of difference of two concave functions. To model this as an MIP we need more 0-1 variables with extra regularity constraints. First let us consider bids submitted by buyers. The bid i with $k(i)$ linear segments is converted into the following $k(i)$ different bids each with a single linear price function.

$$([0, \theta_i^r - \theta_i^{r-1}], \alpha_i^r) \quad r = 1, \ldots, k(i) \tag{12}$$

The contribution of the new bids to the objective function is given by

$$\sum_{r=1,\ldots,k(i)} (\alpha_i^r \sum_{j\in B_i} x_{i(r),j}) \tag{13}$$

where $x_{i(r),j}$ are the new decision variables induced from bid i, which have the same compatibility as i. Let $X_{i(r)} = \sum_{j\in B_i} x_{i(r),j}$ denote the total number of goods traded with bid $i(r)$. The above transformation (12) of bid i into $k(i)$ will be true only if

$$X_{i(r)} > 0 \Rightarrow X_{i(r')} = \theta_i^{r'} - \theta_i^{r'-1} \quad \forall r' < r \tag{14}$$

This condition is satisfied for the above transformation as we are maximizing a concave function P_i. More formally, since $\alpha_i^{r'} > \alpha_i^r$ for $r' < r$ and bid $i(r)$ and $i(r')$ can trade with the same set of offers, we have condition (14) always satisfied when (13) is maximized. The transformation of bid i into $k(i)$ bids given by (12) and (13) introduces no new variables.

However, the same is not true for offers submitted by sellers. This can be readily seen as we are maximizing a convex function (negative of concave function Q_j). On the same lines of transformation for a bid, offer j is converted into $l(j)$ offers, each with a single linear price function:

$$([0, \delta_j^r - \delta_j^{r-1}], \beta_j^r) \quad r = 1, \ldots, l(j) \tag{15}$$

and the contribution of the new bids to the objective function is given by

$$-\sum_{r=1,\ldots,l(j)} (\beta_j^r \sum_{i\in S_j} x_{i,j(r)}) \tag{16}$$

where $x_{i,j(r)}$ are the new decision variables induced from offer j, which have the same compatibility as j. Let $X_{j(r)} = \sum_{i\in S_j} x_{i,j(r)}$ and for the above conversion to hold true we need

$$X_{j(r)} > 0 \Rightarrow X_{j(r')} = \delta_j^{r'} - \delta_j^{r'-1} \;\forall r' < r \tag{17}$$

Since $-\beta_j^{r'} < -\beta_j^r$ whenever $r' < r$, we need extra 0-1 variables and the following constraints for satisfying (17).

$$\begin{gathered}
\textstyle\sum_{i\in S_j} x_{i,j(r)} \leq z_j^r(\delta_j^r - \delta_j^{r-1}) \; r = 1, \ldots, l(j) \\
\textstyle\sum_{i\in S_j} x_{i,j(r)} \geq z_j^{r+1}(\delta_j^r - \delta_j^{r-1}) \; r = 1, \ldots, l(j)-1 \\
z_j^r \in \{0, 1\} \; r = 1, \ldots, l(j)
\end{gathered}$$

3.4 TAX-4

TAX-4 is the general model for a two-attribute exchange with continuous, piecewise linear, concave price functions. Due to the non-zero lower bounds, 0-1 variables and extra constraints are required for both the bids and offers. Bid i in converted into following $k(i)$ new bids.

$$\begin{gathered}
([\theta_i^0, \theta_i^1], \alpha_i^1) \\
([0, \theta_i^r - \theta_i^{r-1}], \alpha_i^r) \quad r = 2, \ldots, k(i)
\end{gathered}$$

The constraints to satisfy (14) are

$$y_i^1\theta_i^0 \leq \quad \textstyle\sum_{j\in B_i} x_{i(1),j} \quad \leq y_i^1\theta_i^1$$

$$\sum_{j\in B_i} x_{i(1),j} \geq y_i^2\theta_i^1$$
$$\sum_{j\in B_i} x_{i(r),j} \leq y_i^r(\theta_i^r - \theta_i^{r-1}) \; r = 2,\ldots,k(i)$$
$$\sum_{j\in B_i} x_{i(r),j} \geq y_i^{r+1}(\theta_i^r - \theta_i^{r-1}) \; r = 2,\ldots,k(i)-1$$

The contribution of bid i to the objective function is

$$y_i^1 m_i + \sum_{r=1,\ldots,k(i)} (\alpha_i^r \sum_{j\in B_i} x_{i(r),j}) \tag{18}$$

Similarly, an offer j with non-zero lower bound in converted into following $l(j)$ offers.

$$([\delta_j^0, \delta_j^1], \beta_j^1)$$
$$([0, \delta_j^r - \delta_j^{r-1}], \beta_j^r) \quad r = 2,\ldots,l(j)$$

The constraints to satisfy (17) are

$$z_j^1\delta_j^0 \leq \quad \sum_{i\in S_j} x_{i,j(1)} \quad \leq z_j^1\delta_j^1$$
$$\sum_{i\in S_j} x_{i,j(1)} \geq z_j^2\delta_j^1$$
$$\sum_{i\in S_j} x_{i,j(r)} \leq z_j^r(\delta_j^r - \delta_j^{r-1}) \; r = 2,\ldots,l(j)$$
$$\sum_{i\in S_j} x_{i,j(r)} \geq z_j^{r+1}(\delta_j^r - \delta_j^{r-1}) \; r = 2,\ldots,l(j)-1$$

The contribution of the offer j to the objective function is

$$-(z_j^1 n_j + \sum_{r=1,\ldots,l(j)} (\beta_j^r \sum_{i\in S_j} x_{i,j(r)})) \tag{19}$$

TAX-2 is a special case of TAX-4 with just one linear segment for all bids. Thus from Theorem 1 it follows that TAX-4 is $\mathcal{NP}$-hard.

4 Multi-attribute Exchanges

In this section we extend the models developed in the previous section for multiple attributes. Let K be the finite set of attributes and let each attribute $k \in K$ can take values from the finite domain. These values need not represent only quantitative values but also qualitative values like color, numerical ranges like frequency range or tolerance range, etc. We say that an attribute value of a bid is compatible with the attribute value of an offer, when they are mutually acceptable. The configurable bids for MAX is $([\underline{b}_i, \overline{b}_i], P_i, U_i^1, \ldots, U_i^K)$ where $U_i^k(e)$ is the price function of attribute k when it takes the value e. The functions P_i and U_i^k are functions of quantity traded but the notation is omitted for clarity (Note U_i^k is a two dimensional function of value e and quantity traded). The contribution of the bid to the objective function is

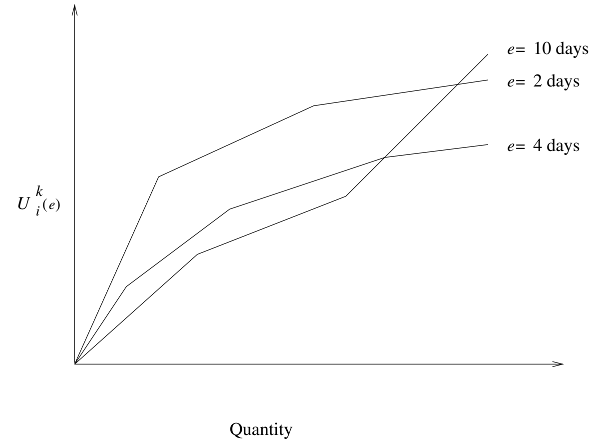

Figure 4. Piecewise linear $U_i^k(e)$ for different values of lead time

$$P_i + \sum_{k\in K} U_i^k(e) \tag{20}$$

The above functional form assumes that the attributes do not have any interaction effects among themselves. We now investigate two different types of U_i^k.

Case 1: Linear function of quantity

In this case, U_i^k has a unit price associated with each of its values but the price is independent of the quantity being traded. Cameras have different markup prices for different warranty periods. There are no volume discounts with such prices and the price function (total price) of such attributes are linear functions of quantity traded. This kind of functions are equivalent to the price markup functions discussed in [5]. Let $\mu_i^{k,e}$ be the slope of the function (unit price markup) of U_i^k when the attribute value is e. Similarly let $\omega_i^{k,f}$ denote the price markup for an offer when the attribute value is f. If the attribute values e and f are compatible, then the surplus generated by trading a single good is given by $\mu_i^{k,e} - \omega_i^{k,f}$. If there are more attribute values that are compatible with each other, then choose the pair that generates the maximum surplus. This preprocessing adds no new variables or constraints to the MIP and only needs some memory to store the compatible value pairs chosen for each bid-offer pair.

Case 2: Continuous piecewise linear function of quantity

Consider the attribute lead time which varies depending on the shipping method used. For each shipping method the price may vary non-linearly with the quantity traded. We assume a continuous, piecewise linear function as shown in Figure 4. The functions need not in general be concave or convex and there may not be common break points among the different attribute values. Let C be the set of compatible value pairs between a bid i and offer j. We need a new binary decision variable w_c which chooses a compatible value pair $c \in C$ and $\sum_{c \in C} w_c = 1$ so that only one value pair is chosen. With suitable modifications, these price functions can be modeled as described in Sections 3.3 and 3.4.

The models considered are not applicable to all markets. The TAX models proposed in Section 3 are applicable to financial markets where a buyer can buy any amount of securities from any seller. This may not be true in other markets. There may be several business rules like restriction on the number of suppliers [5], allowable quantity in a single shipment [8], homogeneity of attributes [5], etc. Adding such business rules as side constraints will involve new decision variables. Furthermore, we have not considered any interaction effects between the attributes. There may be logical restrictions on the allowable combination of the attributes [5]. Incorporating such constraints will scale up the dimensionality of the 0-1 binary variables and requires further investigation.

5 Pricing

Pricing is another important issue in the design of MAX, as it influences the strategic behavior of the traders thereby affecting the outcome of the trade. For example, in VCG pricing [19] mechanisms, traders bid their true value, whereas in midpoint pricing scheme [8], traders have incentive to misrepresent their bids. VCG pricing that dominates the market design literature is not feasible for exchanges as it is not budget-balanced [15, 9], i.e. the money inflow (from the buyers) can be less than the money outflow (to sellers) and the exchange can run at deficit. For practical implementations, budget-balanced pricing schemes are more relevant. Midpoint pricing (buyer pays the midpoint of the surplus to the seller) and pay-your-bid (buyer and seller pay their quoted price) are budget-balanced pricing schemes. Though they do not demand high computational requirements they may require certain regularity conditions while solving the TDP. For example, the midpoint pricing scheme is not applicable to the MIPs presented in Section 3. As pointed in Section 3.2, a buyer may trade with a seller even if they have negative surplus. Applying midpoint pricing scheme may not be favorable to the buyer and/or seller but pay-your-bid pricing scheme still works fine in these cases. Thus the MIPs should be modeled according to the pricing scheme. On the other hand, pricing schemes like value-based pricing [9] and VCG require the TDP to be solved N extra times (for N traders), each time with one trader removed from the market. Efficient algorithmic procedures are required for such pricing schemes to avoid scaling in computational requirements.

6 Conclusions

We proposed MIP formulations for the trade determination problems in MAX with configurable bids. The functional form of the configurable bids considered is the separable additive functions over the attributes, where each function is continuous and piecewise linear. We showed that even for two- attribute exchanges, trade determination is $\mathcal{NP}$-hard for certain bid structures. Related pricing issues were highlighted. Inclusion of business rules as side constraints and allowing logical constraints over the attributes require further investigation. For the exchanges considered in this paper, MIPs for trade determination have a special structure. The linear subproblems, that arises when the 0-1 variables are set, are transportation problems. Due to the simplicity of solving the subproblems, decomposition algorithms like Benders' decomposition [14] and cross decomposition algorithms [18] are good candidates for solving the MIP formulations.

References

[1] M. Bichler, M. Kaukal, and A. Segev, "Multi-attribute auctions for electronic procurement," *First IBM IAC Workshop on Internet Based Negotiation Technologies*, New York, USA, 1999.

[2] M. Bichler, C. Beam, and A. Segev, "An electronic broker for business- to-business electronic commerce on the Internet," *International Journal of Cooperative Information Systems*, 1998, pp. 315-341.

[3] M. Bichler, *The Future of E-Markets: Multi-Dimensional Market Mechanisms*, Cambridge University Press, 2001.

[4] M. Bichler, "BidTaker: An Application of Multi-Attribute Auction Markets in Tourism," *Wirtschaftsinformatik 2002*, Augsburg, Germany, 2002.

[5] M. Bichler, J. Kalagnanam, and H. S. Lee, "RECO: Representation and Evaluation of Configurable Offers," IBM Research Report RC 22288, 2002.

[6] A. Charnes, S. Duffuaa, and M. Ryan, "Degeneracy and the More-for-less Paradox", *Journal of Information & Optimization Sciences*, 1980, pp. 52-56.

[7] J. Kalagnanam, A. J. Davenport, and H. S. Lee, "Computational Aspects of Clearing Continuous Call Double Auctions with Assignment Constraints

and Indivisible Demand," *Electronic Commerce Research*, 2001, pp. 221-238.

[8] S. Kameshwaran and Y. Narahari, "IMPROVE: An Innovative Market for E-Procurement," *International Workshop on Agents for Business Automation*, Las Vegas, USA, June 24-27, 2002.

[9] S. Kameshwaran and Y. Narahari, "A New Approach to the Design of Electronic Exchanges," *Electronic Commerce and Web Technologies*, Lecture Notes in Computer Science 2455, Springer Verlag, 2002.

[10] R. L. Keeny and H. Raiffa, *Decision Making with Multiple Objectives: Preferences and Value Tradeoffs*, Cambridge University Press, 1993.

[11] A. Kothari, T. Sandholm, and S. Suri, "Solving Combinatorial Exchanges: Optimality vis a Few Partial Bids," *AAAI workshop on AI for Business*, 2002.

[12] K. A. McCabe, S. J. Rassenti, and V. L. Smith, "Designing Call Auction Institutions: Is Double Dutch the Best?," *The Economic Journal*, 1992, pp. 9-23.

[13] K. G. Murty, *Linear Programming*, John Wiley & Sons, 1983.

[14] G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization*, John Wiley & Sons, 1988.

[15] D. C. Parkes, J. Kalagnanam and Marta Eso, "Achieving Budget-Balance with Vickrey-Based Payment Schemes in Combinatorial Exchanges." IBM Research Report RC 22218 W0110-065, 2001.

[16] M. Silaghi, "An algorithm applicable to clearing combinatorial exchanges," Technical Report CS-2002-14, Florida Institute of Technology, Florida, 2002.

[17] T. Smith, T. Sandholm, and R. Simmons, "Constructing and Clearing Exchanges Using Preference Elicitation," *AAAI-02 workshop on Preferences in AI and CP: Symbolic Approaches*, 2002.

[18] T. J. Van Roy, "A Cross Decomposition Algorithm for Capacitated Facility Location," *Operations Research*, 1986, pp: 145-163.

[19] R. H. Varian, "Economic Mechanism Design for Computerized Agents," 2000.