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Cost Sharing Mechanisms for Business Clusters with Strategic Firms

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Abstract— **A business cluster is a co-located group of micro, small, medium scale enterprises. Such firms can benefit significantly from their co-location through shared infrastructure and shared services. Cost sharing becomes an important issue in such sharing arrangements especially when the firms exhibit strategic behavior. There are many *cost sharing methods and mechanisms* proposed in the literature based on game theoretic foundations. These mechanisms satisfy a variety of efficiency and fairness properties such as allocative efficiency, budget balance, individual rationality, consumer sovereignty, strategyproofness, and group strategyproofness. In this paper, we motivate the problem of cost sharing in a business cluster with strategic firms and illustrate different cost sharing mechanisms through the example of a cluster of firms sharing a logistics service. Next we look into the problem of a business cluster sharing ICT (information and communication technologies) infrastructure and explore the use of cost sharing mechanisms.**

I. INTRODUCTION

Business clusters refer to co-located groups of micro, small, and medium scale enterprise (MSME) firms engaged in production/delivery of similar types of products/services. Clustering is an important tool in promoting the growth of MSMEs which typically have employees ranging from 10 to about 500. Such firms, in stand alone mode, can lose out to competition in global markets because individual firms cannot achieve economies of scale in production/deliveries. However, these firms can succeed by collaborating with other small firms in a clustered mode.

Firms in the clusters are specialized in the same or complementary phases of the end-to-end supply chain process. Such specialization is considered as one of the key factors in the success of clusters [1]. Each firm can use its limited resources in its core phase of production and the other phases of the production process are taken care of by the other specialized firms in the supply chain. One more important factor that is attributed to the success of clusters in this scenario is cooperation among firms.

Cooperation among firms could be in the form of sharing the cost of information and communication technologies (ICT) infrastructure, sharing the cost of procurement through which the firms can benefit from volume discount offerings, sharing of logistics services, and many more. Firms in a business cluster can benefit significantly through shared infrastructure and shared services. Cost sharing becomes an important issue in such sharing situations especially when the firms exhibit strategic behavior. Since the firms are typically independent, autonomous, rational and intelligent, it is reasonable to expect strategic behavior from the firms. In this paper, we are concerned with suitable cost sharing mechanisms that could be used to share the costs according to best principles of fairness, efficiency, and social welfare.

A. Contributions and Outline

There are a variety of cost sharing mechanisms which have been proposed in game theory and mechanism design literature [2] [3] [4]. These mechanisms satisfy a variety of desirable properties such as allocative efficiency, budget balance, individual rationality, consumer sovereignty, strategyproofness, and group strategyproofness. These cost sharing mechanisms are extremely relevant for business clusters since a cluster comprises rational, intelligent, and autonomous firms. In this paper, we explore the use of such cost sharing mechanisms in the specific context of business clusters. To the best of our knowledge, this is the first time cost sharing mechanisms are being explored in the context of business clusters. The main contributions of this paper are reported in Sections 2 and 3 which are organized as follows.

In Section 2, we fist motivate the problem of cost sharing in a business cluster through the example of a cluster of firms sharing a logistics service. In this example, there are 5 firms which use a single shared logistics service provider. We first introduce the notion of cost sharing methods and then the notion of cost sharing mechanisms. We next introduce desirable properties of cost sharing such as allocative efficiency, budget balance, individual rationality, consumer sovereignty, and incentive compatibility (strategyproofness and group strategyproofness). We next describe relevant cost sharing methods from the literature: (a) Egalitarian cost sharing (b) Average cost sharing (c) Uniform gains or losses (d) Serial cost sharing. We illustrate these cost sharing methods using the logistics services sharing example. Following this, we describe two relevant cost sharing mechanisms, namely the marginal cost pricing mechanism (based on Vickrey-Clarke-Groves mechanisms) and the Shapley mechanism (based on Moulin mechanisms) and again explore these mechanisms for the logistics services sharing example. We then present a more generalized cost sharing scenario in shared logistics services.

In Section 3, we look into a stylized situation of ICT cost sharing in a business cluster consisting of two firms. We use an elegant queueing abstraction to explore cost sharing methods and cost sharing mechanisms for this situation. The

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mechanisms explored here can be extended to the case of a realistic cluster using a queueing network abstraction.

Section 4 concludes the paper with a summary and by presenting several directions in which this work can be enhanced and extended.

II. Cost Sharing Mechanisms and an Example of Logistics Cost Sharing

A. Cost Sharing Mechanisms

A *Cost Sharing Mechanism* is concerned with design of a mechanism in the context of sharing the joint cost of producing a service which is shared among many agents. Given a set N of agents ($N = \{1, 2, ..., n\}$) and their willingness to pay for the service, a cost sharing mechanism determines the set of agents to receive the service.

1) Cost Function C: The cost of producing the joint service is determined by the cost function C. The cost function C is a mapping from $2^N \rightarrow \Re$. Given a subset S of N, $C(S)$ denotes the cost of serving the agents in S. It is assumed that $C(\phi) = 0$.

2) Cost Sharing Method: A *cost sharing method* ξ is a formula that associates to each cost function C and to each coalition S, an allocation of the total cost $C(S)$ among the agents in S i.e., given the cost function and a coalition S, it determines the cost share of each agent. In particular, we denote $\xi_i(S)$ as the cost share of the agent i when the set of agents S is to be served.

A cost sharing method is said to be *cross-monotonic* if adding players to the set S does not increase the cost shares of each agent. For any subset T of S and $i \in T$, $\xi_i(S) \leq \xi_i(T)$

3) Cost Sharing Mechanism: Given the set N of agents willing to receive the service, with each agent having a private willingness to pay for the service, a cost sharing mechanism determines the subset of players to receive the service and sharing of the joint cost among the agents served using the underlying cost sharing method. A cost sharing mechanism invariably uses a cost sharing method. We denote by v_i (valuation) agent i's willingness to pay, x_i as the agent i's cost share allocated by the underlying cost sharing method. The protocol for any cost sharing mechanism can be outlined as follows:

1) Collect bids (b_i) from the agents
2) Based on the bids, determine the subset S of agents to receive the service
3) Determine the price to charge each agent (cost share).

4) Desirable Properties: Ideally a cost sharing mechanisms should satisfy the following properties:

1) *No Positive Transfer* (NPT)
This property requires that the cost shares of the agents determined by the cost sharing mechanism are positive.
2) *Voluntary Participation* (VP)
This property requires that the mechanism should not charge for the agents who are not offered the service.
3) *Consumer Sovereignty* (CS)
This property requires that if an agent reports a high willingness to pay, that agent should be offered service regardless of the other agents' profile.
4) *Budget Balance* (BB)
This condition requires that the set of agents pay exactly the total cost.
5) *Efficiency* (EFF)
The mechanism should maximize the *social welfare*. The term social welfare is defined based on the application and it indicates the welfare that the mechanism is bringing to the society.
6) *Incentive Compatibility*
The two most important notions of incentive compatible mechanisms in the context of cost sharing are *strategyproof*(SP) and *group strategyproof*(GSP).
   - Strategyproof Mechanism: A mechanism is said to be strategyproof if the utility of each player (0 if the agent does not receive the service) should be maximized by bidding truthfully for every fixed set of bids by the other agents.
   - Group Strategyproof Mechanism: A mechanism is said to be group strategyproof if no coordinated false bid by a subset of players can ever strictly increase the utility of one of its members without strictly decreasing the the utility of some other player.

B. A Motivating Example: Logistics Services

Fig. 1. Logistics Services Sharing

As a motivating example of the cost sharing model, we consider an example of shared logistics services in business clusters. This example is based on a mail distribution example presented in [5]. Figure 1 shows the example. There are five firms named A, B, C, D and E. A logistics provider located at Ω provides logistics services to the five firms through a fleet of vehicles. The logistics provider's charge is \$1 per kilometer of the vehicle travelled.

As shown in figure 1, all the firms are physically located along the single lane starting at Ω and passing successively through A, B, C, D and E. Distances in kilometers, are indicated in the figure. Thus the daily tour from Ω to E and back costs \$110 assuming that all the firms can be served in one round trip. Throughout this paper, we will assume that all the firms can be served in one trip. Each firm has some willingness to pay for the logistics service. They submit this as a bid to the service provider (or mechanism designer). The problem here is to divide the total cost (\$110) among all five firms such that the cost share of each firm is less than or equal to the bid of that firm. Thus a cost sharing mechanism takes all these as inputs and determines which firms get service and what will be their cost shares.

C. Cost Sharing Methods for Shared Logistics

Several cost sharing methods have been proposed in the literature [6],[2]. We will look at some of those in this paper.

1) Egalitarian Cost Sharing Method: Egalitarian cost sharing method divides the total cost of providing the service among the receiving agents equally. If S denotes the set of agents who receive the service, cost shares of the agents is given by the formula

$$\xi_i(S) = \frac{C(S)}{|S|} \; \forall i \in S$$

In the logistics services example of Figure 1, if we take S as all the five firms, the total cost of providing the service to S i.e., $C(S)$ is \$110, and the cost shares are $x_i = \frac{110}{5} = \$22$.

2) Average Cost Sharing: Average cost sharing method shares the cost proportionally among the agents. If the set S of agents is to be served, then the average cost sharing method shares the cost $C(S)$ proportionally among the agents in S. Hence the cost shares of the agents in S are given by

$$\xi_i(S) = \frac{C(S)}{D} \times d_i$$

where $D = \sum_{i \in S} d_i$, d_i is a parameter that is proportional to the standalone cost of providing service to i.

In the logistics services example, the parameter d_i could be taken as twice the distance from the preceding firm. If all the firms are to be served, then for firm A, $d_A = 20$ because it is 10 units away from the logistics provider, hence the cost share of firm A is $\frac{110}{110} \times 20$ i.e., $x_1 = \$20$. Similarly the cost shares of the firms B,C,D and E are \$10, \$60, \$10, \$10 respectively.

3) Uniform Gains or Losses: Uniform gains or losses method shares profit or loss incurred by cooperation equally among the agents in the set S.

In the logistics services example, if the set S is taken as all the five firms, then $C(S) = \$110$, but if the service is provided to each firm separately, the costs would be \$20, \$30, \$90, \$100, and \$110 respectively. Hence the total profit gained if all the firms in the set S cooperate would be $(20 + 30 + 90 + 100 + 110) - 110 = \$240$. Uniform gains or losses method requires that this profit be shared among all the five firms equally. Hence the cost share of firm A would be $20 - \frac{240}{5} = -\$28$. Negative cost share indicates that the service provider should give \$28 to firm A. Similarly cost shares for firms B,C,D and E are $-\$18$, \$42, \$52, and \$62 respectively.

4) Serial Cost Sharing: Serial cost sharing method attributes the cost of the distance travelled so far, to all the firms that are to be served after travelling this distance.

Suppose that a_1, a_2, a_3,... are the firms in the order in which they are arranged, away from the distribution system present at the left end before a_1. Let $C(a_i)$ denote the cost of providing service to the set $\{a_1, a_2, ..., a_i\}$ of agents, then the cost shares are given by

$$x_1 = \frac{C(a_1)}{n}$$
$$x_2 = x_1 + \frac{C(a_2)-C(a_1)}{n-1}$$
$$x_3 = x_2 + \frac{C(a_3)-C(a_2)}{n-2}$$

For the logistics services example, the cost shares using serial cost sharing would be $x_A = \frac{20}{5} = \$4$, $x_B = 4 + \frac{30-20}{4} = \$6.5$. Similarly, $x_C = \$26.5$, $x_D = \$31.5$, and $x_E = \$41.5$.

D. Cost Sharing Mechanisms for Shared Logistics

The literature on strategyproof mechanisms when preferences are quasi-linear emphasizes the tradeoff between efficiency and budget balance: a strategyproof mechanism can be efficient (in shared logistics model: serve the joint surplus maximizing set of users) or it can balance the budget (in shared logistics model: the total payment collected from the firms covers the total cost exactly) but it cannot be both efficient and budget balanced [7],[8].

As a consequence of this result, two alternatives are possible, either we can compromise on balancing the budget or compromise on efficiency. *Moulin Mechanisms* [2] [9] come under the the budget balanced category of cost sharing mechanisms, they need not be efficient. *Vickrey-Clarke-Groves Mechanisms* [10],[11], [12] come under the category of cost sharing mechanisms which are efficient but need not be budget balanced.

Moulin mechanisms have the property that they are strategyproof if the underlying cost sharing method is cross-monotonic [3].

Vickrey-Clarke-Groves (VCG) Mechanisms, the class of strategyproof and efficient mechanisms that we consider here have the property that there is only one mechanism in that class called *Marginal Cost Pricing mechanism* that satisfies both NPT and VP [3].

1) Marginal Cost Pricing Mechanism: Marginal cost pricing mechanism belongs to the class of VCG mechanisms. The general scheme of any VCG mechanism is the following:

1) define "social welfare"
2) find the set of players that optimize the social welfare
3) compute the optimal social welfare when a player joins the game, and when he does not join the game
4) the player should be charged such that his individual welfare is the increase he brings to the social welfare

For the marginal cost mechanism, we will define social welfare as $w(S, b) = max_{T \subseteq S} [b_T - C(T)]$, where $b_T = \sum_{i \in T} b_i$. A coalition is said to be *efficient* if

$$b_S - C(S) = w(N, b)$$

Marginal cost pricing mechanism selects the largest efficient coalition S as the winning coalition and this set S is offered the service. We use a binary variable q_i to denote if player i is in the winning coalition or not, if $q_i = 1$ then the player i is in the winning coalition otherwise he is not in the winning coalition. Cost shares are given by the following equation:

$$x_i = b_i * q_i(b) - (w(N, b) - w(N - i, b)) \qquad (1)$$

The term $w(N - i, b)$ denotes the optimal social welfare when the player i does not join the game and $w(N, b)$ when the player joins the game. Hence the player i is given the discount that is equal to the increase $w(N, b) - w(N - i, b)$ that he brings to the social welfare. This fact is reflected in the cost shares equation given in equation 1.

Marginal cost pricing mechanism meets *NPT*, *VP*, and *CS* and is strategyproof [3], but is not group-strategyproof [3].

As the VCG mechanisms are not budget balanced, Moulin and Shenker proved that among the VCG class of mechanisms, marginal cost pricing mechanism has the lowest budget loss.

For the logistics services example, if we consider the firms' bids as $\frac{2}{3}^{rd}$ of their standalone costs, we get $b_A = 15$, $b_B = 22.5$, $b_C = 67.5$, $b_D = 75$, $b_E = 82.5$. On this bid profile, we take the social welfare as

$$w(S, b) = \max_{T \subseteq S} \{b_T - C(T)\}$$

For a set S of agents, this equation computes the maximum surplus that can be generated by any subset of the agents.

Using this as the social welfare, the set S of agents chosen by marginal cost pricing mechanism is $S = \{A, B, C, D, E\}$ and their cost shares $x_A = \$0$, $x_B = \$0$, $x_C = \$0$, $x_D = \$0$, $x_E = \$10$. It can easily be observed that the total cost of providing service to S is $C(S) = \$110$ where as marginal cost pricing mechanism is collecting only \$10 which is not budget balanced.

2) Shapley Value Mechanism: Shapley value mechanism belongs to the class of mechanisms called Moulin Mechanisms. In general, Moulin mechanisms run in several iterations. In each iteration it refines the set of agents to receive the service. Shapley value mechanism is a Moulin mechanism with the underlying cost sharing formula as the Shapley value formula.

The Shapley value formula is given by the equation,

$$x_i(S) = \sum_{T \subseteq S - i} \frac{|T|!(|S| - |T| - 1)!}{|S|!} [C(T \cup i) - C(T)] \quad (2)$$

Using this cost sharing formula, the Shapley value mechanism is built. The mechanism is as follows:

1) Set $S = N$
2) Determine the cost shares $x_i(S)$ for all i using the Formula 2
3) Ask each user i if the cost share x_i is less than the willingness to pay b_i
4) Remove all the users who say no to the above question from S
5) Repeat steps 2,3,4 until all the players in set S say YES

After the completion of all the iterations, the set S is the winning coalition.

Moulin and Shenker [6], showed that among all the mechanisms derived from cross-monotonic sharing methods, Shapley value mechanism has the smallest maximal efficiency loss. In contrast to this proposition, Mutuswami [4] showed that the egalitarian cost sharing method is characterized by the property that the expected size of the coalition receiving service is maximal.

Shapley value mechanism and in general, Moulin mechanisms, satisfy NPT, VP, CS and strategyproof and they are budget balanced. But they are not efficient.

For the logistics services example, with the bids and cost function as defined for marginal cost pricing mechanism, set of firms to offer the service is $S = \{A, B, C, D, E\}$, and their cost shares are $x_A = \$4$, $x_B = \$6.5$, $x_C = \$26.5$, $x_D = \$31.5$, $x_E = \$41.5$.

Based on the cost shares computed by all the cost sharing mechanisms explored here, we can see that VCG mechanisms are not budget balanced where as Moulin mechanisms are budget balanced for our example.

E. A Generalized Scenario for Shared Logistics Services

In the logistics services example, we have considered that all the firms in the cluster lie on a single lane. This is highly unrealistic. If we consider a realistic scenario, the firms in the cluster may spread through the cluster. For example, consider Figure 2, which shows a small business cluster with four firms A, B, C and D and a service provider O.

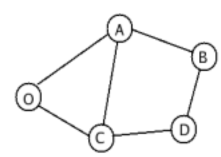

Fig. 2. Logistics Cost sharing in Business Clusters

The firms can be imagined as nodes in the graph, the paths connecting two firms as the edges connecting the corresponding nodes. Logistics provider can now be viewed as an extra node. Each edge has cost associated with it. The charge of the carrier is proportional to the distance travelled. Now, this problem is similar to the *Metric Traveling Sales Person (TSP) game*, in which edge costs between nodes satisfy the triangular inequality, and the travelling salesman starts at node 1 and executes a tour, visiting a set of users that are chosen by the cost sharing mechanism. Jain and Vazirani [13], propose a cost sharing mechanism for metric TSP game which is 2-approximation budget balanced, group strategyproof. This mechanism is based on the fact that doubling an Minimum Spanning Tree (MST), finding an Eulerian tour and short cutting gives a factor 2-approximation algorithm for metric TSP.

III. Information and Communication Technology Cost Sharing

Business clusters, as characterized by the geographical locality of all the firms, can benefit by sharing the cost of establishing *Information and Communication Technology* (ICT) within the cluster. ICT applications like email, websites, networking, etc are the basic applications which can ensure effective business communication and increase the visibility of business clusters in the global markets. But not all firms can afford ICT, hence all the firms in the cluster can cooperate for establishing ICT infrastructure and they can share the joint cost for this service. Each firm in the business cluster will benefit from the shared infrastructure.

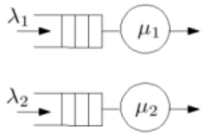

Fig. 3. Firms in a cluster without shared infrastructure

Consider a stylized example of a business cluster consisting of two individual firms. Currently the two firms have separate stand alone Internet infrastructure consisting of a Internet up-linking facility with routers and other relevant switches. The Internet traffic at firm i $(i = 1, 2)$ can be approximated as a Poisson process with arrival rate λ_i $(i = 1, 2)$ while the Internet service process at each firm can be modeled as independent exponential random variables having rates μ_i $(i = 1, 2)$. Figure 3 depicts a simple queueing model for this scenario. Mean end-to-end delay is given by the formula

$$D_i = \frac{1}{\mu_i - \lambda_i}$$

Suppose, $\mu_1 = 5$, $\lambda_1 = 4$ for firm 1 and $\mu_2 = 10$, $\lambda_2 = 8$ for firm 2. Mean end-to-end delay for firm 1 and firm 2 are $D_1 = 1$ and $D_2 = \frac{1}{2}$ respectively.

Since the traffic in both the firms is heavy, the two firms face long delays in Internet responses affecting the productivity of their employees. The two firms decide to invest on a shared high speed Internet infrastructure with the agreement that approximately half the traffic will be routed to the newly created shared infrastructure. The new situation is modeled in Figure 4. Mean end-to-end delay ([14] gives more details on performance modeling) in this scenario for firm 1 is given by

$$D_1' = \frac{1}{2} \times \left\{ \frac{1}{\mu_1 - \frac{\lambda_1}{2}} \right\} + \frac{1}{2} \times \left\{ \frac{1}{\mu_s - \left(\frac{\lambda_1}{2} + \frac{\lambda_2}{2}\right)} \right\}$$

Similarly, D_2' can be computed. Clearly, the newly created infrastructure will lead to much faster Internet connectivity for both the firms, leading to increased productivity.

Now with the numerical example that we took, if $\mu_s = 20$, the new delay offered by firm 1 is $D_1' = \frac{17}{84}$ and $D_2' = \frac{5}{42}$. Hence it can be observed that both the firms now benefit from the shared infrastructure by offering reduced delays to its arrivals but the benefits are not equal.

A key question to be answered here is how does the two firms share the cost of creating the new infrastructure in a way that is agreeable to both the firms. We will investigate this question using cost sharing mechanisms.

We measure the amount of benefit that a firm gets from the shared infrastructure by Δ_i, called the *benefit measure*. In the queuing example that we considered, Δ_1 is the difference between the old delay D_1 and the new D_1' and $\Delta_1 \geq 0$.

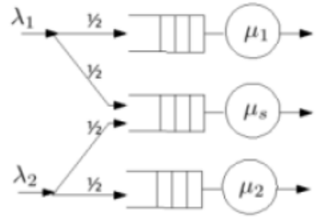

Fig. 4. Firms in a cluster with shared infrastructure

$$\Delta_1 = D_1 - D_1'$$

$$\Rightarrow \Delta_1 = \frac{2\mu_1\mu_s - \lambda_1^2 - 2\mu_1^2 + 2\lambda_1\mu_1 - \mu_1\lambda_2}{(\mu_1 - \lambda_1)(2\mu_1 - \lambda_1)(2\mu_s - \lambda_1 - \lambda_2)} \quad (3)$$

For our example, $\Delta_1 = \frac{67}{84}$ and $\Delta_2 = \frac{16}{42}$ for $i = 1$ and $i = 2$ respectively. The problem that we are addressing here is how to share the joint cost of this infrastructure among the firms given that each firm has its own willingness to pay for the service and each firm benefits from the infrastructure by different amounts.

A. Cost Function C

We assume that the cost function takes the following form $C(S) = \sum_{i \in S} C_i + C_k$, where C_i is the cost incurred for infrastructure at firm i, and C_k is the fixed cost that is incurred for shared infrastructure. We assume that $\forall i$, $C_k >> C_i$.

For the numerical example that we considered, if $C_1 =$ \$100, $C_2 =$ \$120 and $C_k =$ \$600. As a result, $C(\{1\}) = 700$, $C(\{2\}) = 720$, $C(\{1, 2\}) = 820$.

B. Cost Sharing Methods

1) Proportional Cost Sharing: If we impose the constraints that the cost share of firm i should be directly proportional to the benefit measure Δ_i and the cost sharing method should cover the cost of serving the firms, proportional cost sharing method based on benefit measure Δ_i can be used.

$$\xi_i(S) = \frac{\Delta_i}{\Sigma_{j \in S} \Delta_j} C(S) \quad (4)$$

Cost sharing method defined in equation (4) need not be cross-monotonic. For the numerical example that we considered, if we take $S = \{1, 2\}$, then $x_1(S) = 554.95$ and $x_2(S) = 265.05$. It can be observed that the cost shares given by this cost sharing method are proportional to the benefit measure.

2) Serial Cost Sharing: For the numerical example that we considered, we know that $C(\{1\}) = 700$, $C(\{2\}) = 720$ and $C(\{1, 2\}) = 820$. If we arrange the set of firms in the increasing order of their costs, we get $S = \{1, 2\}$ (we consider costs of only singleton sets while ordering). Serial cost sharing method gives

$\xi_1(S) = \frac{C(\{1\})}{2} = 350$
$\xi_2(S) = \xi_1(S) + \frac{C(\{1,2\}) - C(\{1\})}{2-1} = 350 + 120 = 470$

It can be noted that, serial cost sharing method purely shares the cost based on the cost function. It does not take the benefit measure into account. Firm 2 is charged more because it costs more to provide service to this firm.

3) Shapley Value: Applying Shapley value given in equation (2) to the cost function defined above with $S = \{1, 2\}$, we get $\xi_1(S) = 400$, $\xi_2(S) = 420$.

It can be noted that even Shapley value shares entirely based on cost function. This does not take benefit measure into account. Moulin and Shenker showed [3] that Shapley value cost sharing method belongs to the class of cross-monotonic cost sharing methods.

C. Cost Sharing Mechanisms

For ICT cost sharing problem, we consider only Moulin mechanisms and Marginal cost pricing mechanism.

1) Moulin Mechanisms: For every cross-monotonic cross sharing method, ξ, Moulin and Shenker ([3], [15]) gave the following mechanism $M(\xi)$ which computes the set S of firms to offer the service and their cost shares

Mechanism $M(\xi)$

1) Initialize $S = N$
2) If there is a user $i \in S$ such that $b_i < \xi_i(S)$ then remove i from S. Repeat this step until there is no such user i for which $b_i < \xi_i(S)$
3) Set $x_i = \xi_i(S)$

Moulin and Shenker also proved ([3], [15]) that for any cross-monotonic cost sharing method ξ, the mechanism $M(\xi)$ is *budget balanced*, meets *NPT*, *VP*, *CS*, and is *group strategyproof*.

Hence, Shapley value mechanism is budget balanced, and it meets NPT, VP, CS, and is group strategyproof. But it need not satisfy EFF. But if the service provider wants to charge more to the firm with more benefit, then Moulin mechanism can be used with proportional cost sharing method. This mechanism is budget balanced, NPT, VP, CS but it need not be group strategyproof.

2) VCG Mechanisms: As already stated, among the class of VCG mechanisms, marginal cost pricing mechanism has the lowest budget loss. Hence we consider only this mechanism.

With the bids for firms 1 and 2 as $b_1 = 500$ and $b_2 = 400$, marginal cost pricing mechanism selects the set $S = \{1, 2\}$ and gives $x_1 = \$320$ and $x_2 = \$220$. It can be seen that the budget loss in this case is \$280. Marginal cost pricing mechanism meets NPT, VP, CS and is strategyproof but not group strategyproof.

Because marginal cost pricing mechanism is not budget balanced, this can be used when an external financial agency is willing to fill the budget loss.

Hence the service provider is provided with a choice of parameters based on which appropriate cost sharing mechanism in the context of business cluster can be used.

IV. Summary and Future Work

A business cluster is a co-located group of micro, small, medium scale enterprises. Such firms can benefit significantly from their co-location through shared infrastructure and shared services. Cost sharing becomes an important issue in such sharing arrangements. In this paper, we have investigated the problem of cost sharing in a business cluster through two representative examples. In the first example, we looked into a cluster of firms sharing a logistics service. In the second example, we investigated the problem of firms in a business cluster sharing ICT (information and communication technologies) infrastructure. We have explored the use of cost sharing mechanisms for these situations and described several desirable properties satisfied by these mechanisms.

To the best of our knowledge, this is the first time cost sharing mechanisms proposed in the game theory and mechanism design literature have been adopted to cost sharing problems in business clusters. Since the firms in a business cluster are independent, autonomous, rational, and intelligent entities, such cost sharing mechanisms are highly appropriate.

We have only considered stylized examples in this paper. Both the situations investigated, namely logistics service sharing and ICT infrastructure sharing, need to be enhanced and extended to realistic situations. Some ideas have already been provided in the respective sections but much remains to be done.

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