



TRANSIENT ANALYSIS OF MULTICLASS MANUFACTURING SYSTEMS WITH PRIORITY SCHEDULING

Y. Narahari†‡, N. Hemachandra§ and M. S. Gaur¶

Department of Computer Science and Automation, Indian Institute of Science, Bangalore-560012, India

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Scope and Purpose—Markov chains constitute the basic stochastic model underlying most performance evaluation studies in the area of manufacturing systems. Analysis of Markovian models has traditionally emphasized steady-state or equilibrium performance in preference to transient performance. Several studies over the years have, however, shown that transient analysis is also relevant. Most such studies have been carried out under assumptions of no setup or switchover times and simple FCFS scheduling. This article examines the importance of transient analysis, by studying in detail, a typical manufacturing facility that involves the production of several classes of products belonging to different priority levels and involving significant switchover times. The motivation of the study is to investigate the transient performance of a model that incorporates two real-world features namely switchover times and priority scheduling. Our numerical experiments confirm that the transient performance of the system over typical observation intervals can be significantly different from the performance predicted by steady-state analysis. In such systems, transient analysis will have important implications for decision making during design and operation.

Abstract—In this article we examine the transient performance of a flexible machine centre that processes several classes of jobs with significant setup times and with priority scheduling. Our results show that the transient performance of the system over typical observation intervals can be significantly different from the performance predicted by steady-state analysis. The machine centre serves three classes of jobs—class 1, class 2 and class 3; class 3 jobs have non-preemptive priority over jobs of class 1 and class 2. The results show that the throughput and cycle time of class 1 and class 2 jobs are affected quite dramatically by the arrival of class 3 jobs. However, over a typical observation period in the initial evolution of the system, steady-state results overestimate this effect of the high priority jobs whereas the more credible estimates are given by transient analysis. The transient analysis is carried out by solving explicitly the corresponding Markov chains, using a higher level stochastic Petri net model to generate the Markov chains. © 1997 Elsevier Science Ltd. All rights reserved

1. INTRODUCTION

Studies in performance evaluation of discrete manufacturing systems have traditionally emphasized steady-state or equilibrium performance in preference to transient performance [1–3]. Recently, studies have shown that transient analysis is extremely relevant in several manufacturing system analyses [4]. However previous studies on transient analysis of manufacturing system models have only considered models with simple FCFS scheduling and without setup or switchover times. In this article, we study the transient performance of multiclass manufacturing systems with significant switchover times, under priority scheduling. In particular, we focus on the performance of a multiclass production facility by

† To whom all correspondence should be addressed.

‡ Currently, Dr Yadati Narahari is an Associate Professor at the Department of Computer Science and Automation, Indian Institute of Science, Bangalore, India. He completed his Master's in Computer Science and his Ph.D., from the same department. Collaborating with Professor Viswanadham at the same department, Dr Narahari completed a comprehensive text entitled *Performance Modeling of Automated Manufacturing Systems*. He has written papers in *IEEE Transactions on Robotics and Automation*, *IEEE Transactions on Semiconductor Manufacturing*, *Annals of Operations Research*, *Journal of Manufacturing Systems*, *Computers & Operations Research and Advances in Petri Nets*. More recently, Dr Narahari has been interested on object oriented analysis and design in the context of software development for manufacturing systems.

§ N. Hemachandra received the B.E. degree in Electrical Engineering from the Jawaharlal Nehru Technological University, Kakinada, India, and his M.E. Degree in Electrical Engineering from the Indian Institute of Technology, Kharagpur, India, in 1989 and 1991, respectively. He was a doctoral candidate at the Department of Computer Science and Automation, Indian Institute of Science, Bangalore during 1991–96. His doctoral dissertation was on the analysis and optimization of queueing models with correlated input arrivals. Currently, he is a research engineer with Siemens Information Systems Ltd, Bangalore, and working on manufacturing systems logistics, supply chain and scheduling software. His current research interests are in the areas of queueing theory, scheduling methodologies and stochastic modelling, applied to manufacturing systems and data networks.

¶ M.S. Gaur received the B.E. degree in Electronics from the Regional College of Engineering at Jaipur, Rajasthan, India and the Master of Engineering Degree in Computer Science from the Indian Institute of Science, Bangalore, India in 1994. He is currently working as an Assistant Professor in the Department of Computer Science at the Regional Engineering College, Jaipur. His areas of interest include performance modelling and analysis of computer systems, design of data networks, and object oriented analysis and design.

conducting transient analysis of its Markovian model and show that transient performance estimates can be significantly different from steady-state analysis estimates.

In this article, we view a manufacturing system as a discrete event dynamical system and consider that the evolution of a manufacturing system constitutes a discrete state space process [3]. In particular, we focus on Continuous Time Markov Models. Such a model could be generated directly or by using higher level models, such as Queueing Networks, Stochastic Petri Nets, or Discrete Event Simulation [3].

Steady-state analysis has been the focus of most performance studies in the area of manufacturing systems. The main reasons for this popularity of steady-state analysis include:

- Major results in queueing theory are all concerned with steady-state analysis—see for example, [3].
- There are many computationally efficient and simple methods for steady-state analysis [5].
- Developments in aggregation and decomposition methods aimed at large state space, real-world problems have also focused on steady-state analysis.

Often discrete event system models do not have steady state or do not reach steady state in an observation period of interest. Transient analysis becomes important in such situations. There is a vast amount of literature on transient analysis of Markovian models. The classic early work of Morse [6] touches upon many computational aspects of transient analysis. There are many other early and recent articles of relevance, including: Grassmann [7–9], Whitt [10], Muppala and Trivedi [11], and Reibman and Trivedi [12], to name a few. The relevance of transient analysis to manufacturing system models has been recently enunciated by Narahari and Viswanadham [4]. Some important problems in manufacturing which need transient analysis are:

- Analysis of manufacturing systems with nonstationary workloads [13].
- Reliability and performability studies [14].
- Performance analysis over finite observation periods [4].
- Performance under unstable scheduling policies [4].

In most of the existing studies, the models investigated are either single class models without setup times or multiclass models without setup or switchover times. Also, usually, a simple FCFS scheduling policy is assumed. In [4], an example of a machine centre that produces two types of products is presented and it is shown that transient analysis yields performance values that are significantly different from those obtained using steady-state analysis. In this article, we discuss a more detailed example that involves a priority job class in addition to other job classes and carry out a detailed transient analysis of the system. The motivation of the study is to investigate the transient performance of a model that incorporates two real-world features, namely switchover times and priority scheduling. Two different types of priority job classes are considered and the analysis is done independently in each case. In case 1, the priority job class comprises a stream of slowly-arriving jobs with large processing times (call them Slow, Long, Priority jobs or SLP jobs for short). In case 2, the priority job class comprises a stream of rapidly arriving jobs with low processing times (call them Fast, Short, Priority jobs or FSP jobs for short). Our results show that:

- (1) Transient performance values of throughput and cycle time over a finite observation period are much different from steady-state values. In fact, steady-state values underestimate the throughput and overestimate the cycle time of the nonpriority jobs.
- (2) Both SLP and FSP jobs affect quite dramatically the throughput and cycle time of nonpriority jobs.

In the context of the multiclass manufacturing cell that we investigate in this article, the transient analysis facilitates a better understanding of the system behaviour and also leads to more credible performance analysis. Besides, it has important design implications such as:

- (1) We can use the expected manufacturing lead times over finite operation intervals to set meaningful due dates for low priority and high priority jobs.
- (2) We can estimate the capacities of input buffers so as to assure certain minimum performance levels in terms of minimum throughput rate and maximum lead time, etc.
- (3) Since performance measures over finite intervals do depend on the initial state of the system, we can determine an appropriate initial state that ensures the required performance level to be achieved.

- (4) Since the presence of high priority jobs severely limits the production of low priority jobs due to the combined effect of priorities and switchovers, one can determine an upper bound on the arrival rate of high priority jobs that would guarantee a minimum level of production level for low priority jobs.

The above issues can also be addressed using steady-state analysis. However, over finite observation periods, since transient analysis is more credible, we would get more accurate answers for our questions using transient analysis.

In Section 2, the manufacturing facility investigated is discussed in detail and the performance measures of interest are defined. Section 3 looks at the effects of slow, long priority jobs, while Section 4 has the details of the effects of fast, small priority jobs. Section 5 is the conclusion.

2. A MULTIPRODUCT MANUFACTURING FACILITY

A machine centre with a flexible or versatile machine that can process a variety of part types is an essential building block of flexible manufacturing or assembly systems. Several studies in the literature have explored performance and scheduling issues connected with flexible machine centres. See, for example, the books by Viswanadham and Narahari [3], Buzacott and Shantikumar [1], and Gershwin [2]. In this article, we carry out detailed investigations on a flexible machine centre producing three types of products.

Figure 1 gives the schematic. Let the nonpriority jobs be called type 1 and type 2 jobs, whereas the priority jobs be called type 3 jobs. The machine switches production among these products by giving non-preemptive priority to type 3 jobs and takes up type 1 or type 2 jobs only when there are no type 3 jobs waiting for production; it takes up type 1 jobs when there are no type 2 jobs and *vice versa* (in the case of both type 1 and type 2 waiting it chooses type 1 or type 2 with probability 0.5 each). In contrast, while serving type 1 or type 2 jobs (with no type 3 jobs waiting), it follows an exhaustive policy, i.e. once set up for a particular type, say type 1, processing is done on all type 1 parts until no more type 1 parts are waiting in the buffer. Finally, the processing of all the jobs is non-preemptive.

The machine will become idle when there is no part waiting to be produced. We assume that there is a certain switchover or setup time involved when the machine takes up a new type of part for production. The buffers (buffer 1, buffer 2, and buffer 3) shown in the schematic are assumed to have capacities N_1 , N_2 , and N_3 , respectively. Finite capacities for the buffers are assumed to ensure finiteness of the state space and amenability to numerical experimentation. The items in buffer i ($i=1, 2, 3$) could correspond to one of the following:

- Raw parts of type i waiting for their turn to get processed by the machine. In this case, exogenous arrivals into buffer i correspond to externally arriving raw parts of class i .
- External demands for class i products. In this case, exogenous arrivals into buffer i correspond to arriving external demands for class i products. Here, raw material is assumed to be always available and the machine will consider a class for production only if demands are outstanding in the buffer.

2.1. Operating rules and assumptions

In the discussion that follows, we assume the first of the above interpretations; the discussion is equally valid and relevant for the other interpretation. We make the following assumptions about the operation

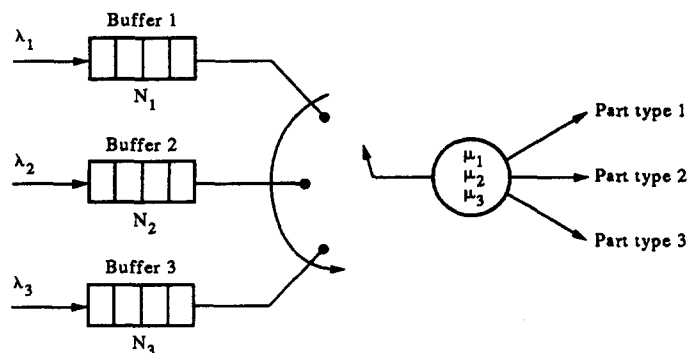


Fig. 1. A multiproduct manufacturing facility.

of the system:

- (1) Raw parts of class i arrive into the system according to a Poisson process with rate λ_i . Arriving raw parts of type i that find buffer i full leave the system without undergoing service. The setup time for type i , which is also the time to switch over from another type to type i is a stochastic variable distributed exponentially with rate s_i . The processing times for type i jobs are exponentially distributed with rate μ_i . All the random variables involved are independent.
- (2) The type 3 jobs get non-preemptive priority over type 1 and type 2 jobs. That is, when a new type 3 raw part arrives into buffer 3 while the machine is serving the nonpriority parts, the machine finishes the current part being processed and switches over to produce type 3 part. As long as the system has no type 3 parts to work on, it will service type 1 and type 2, following the exhaustive policy. That is, for example, if the machine centre is currently set up for type 1, it will process type 1 until no more of type 1 parts are available for processing and then switch over to type 2 if type 2 parts are available.
- (3) After exhausting the service of type 3 parts, if the machine finds both type 1 and type 2 parts waiting, it will switch over to type 1 or type 2 with equal probability.
- (4) FCFS (*First Come First Served*) policy is used for dispatching parts in a given buffer.
- (5) The machine does not fail during the interval of observation.

2.2. Performance model

Under the assumptions above, the model of the above system is a continuous time Markov chain. The Markov chain can be generated easily from a higher level stochastic Petri Net model [15,3]. In fact the SPNP (Stochastic Petri Net Package) [15] tool was used to analyse the model. The construction of a stochastic Petri Net model for the above system in itself is an interesting task. The detailed Petri Net model is discussed in [16]. The state space of the Markov chain depends on the capacities of buffer 1, buffer 2, and buffer 3. The performance model is described by the following parameters: N_1, N_2, N_3 (buffer capacities); $\lambda_1, \lambda_2, \lambda_3$ (arrival rates); μ_1, μ_2, μ_3 (service rates); and s_1, s_2, s_3 (setup rates).

2.3. Performance measures

We consider the following performance measures for the above system: the first two are transient measures, whereas the next two are their steady-state counterparts.

- Average cycle time or average manufacturing lead time (MLT) of type i jobs during $[0, t]$. The MLT of a job is the total time the job spends waiting and getting serviced in the system.
- Average accumulated throughput of type i parts during an interval $[0, t]$. This represents the average total number of parts of type i produced during $[0, t]$. Here, t can be any period of observation, or a full day's operation, etc.
- Mean steady-state MLT of type i jobs.
- Mean steady-state throughput rate or production rate for type i jobs.

To define the above performance measures precisely, let $\{1, 2, \dots, K\}$ be the state space of the underlying markov chain. We associate appropriate reward rates with these states, as in [11]. For $i = 1, 2, \dots, K$, let r_i be the reward rate in state i . Suppose x represents the random variable corresponding to the reward rate in the steady state, then the expected reward rate $E[X]$ can be computed as:

$$E[X] = \sum_{i=1}^K r_i \pi_i,$$

where π_i is the steady-state probability of state i . The expected value of the transient or time-dependent reward rate is defined by

$$E[X(t)] = \sum_{i=1}^K r_i P_i(t),$$

where $P_i(t)$ is the probability of state i at time t . The expected value of the accumulated reward, $Y(t)$, over time interval $[0, t]$ can be computed as

$$E[Y(t)] = \sum_{i=1}^K r_i \int_0^t P_i(u) du$$

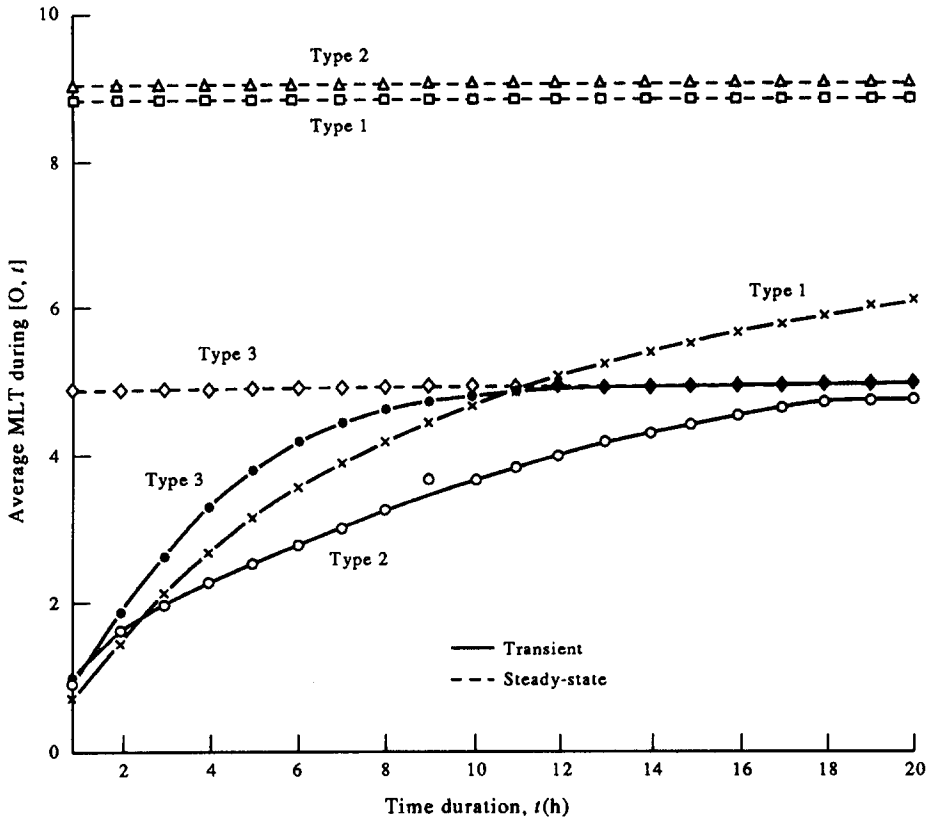


Fig. 2. Variation of MLT of SLP and nonpriority jobs.

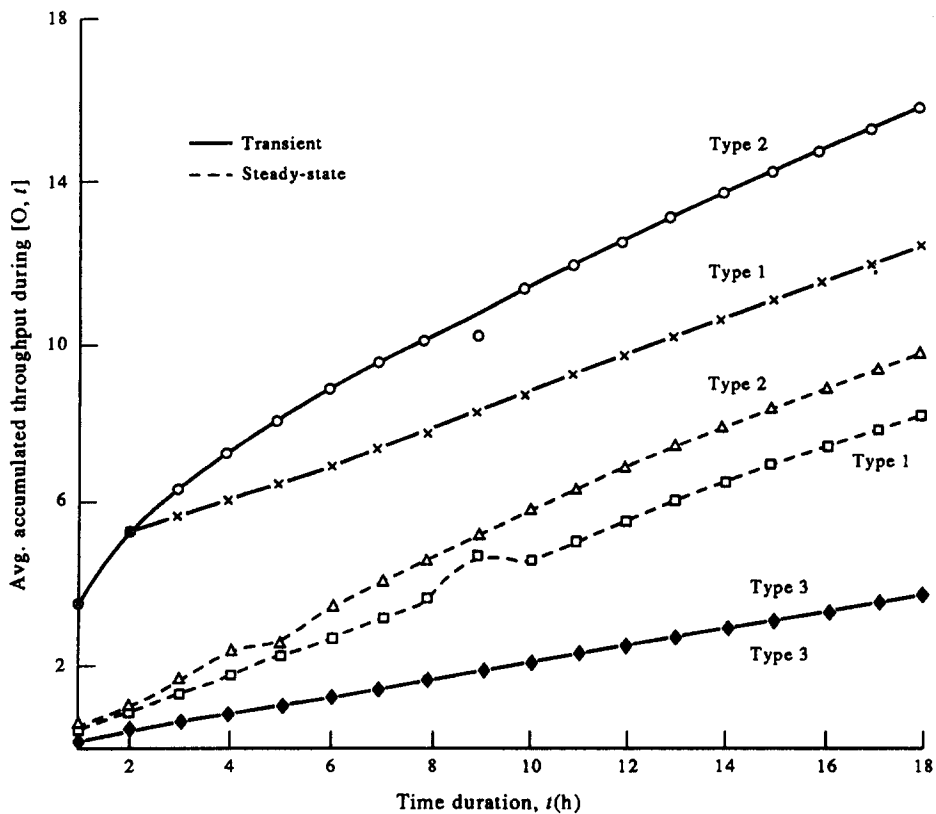


Fig. 3. Variation of throughput of SLP and nonpriority jobs.

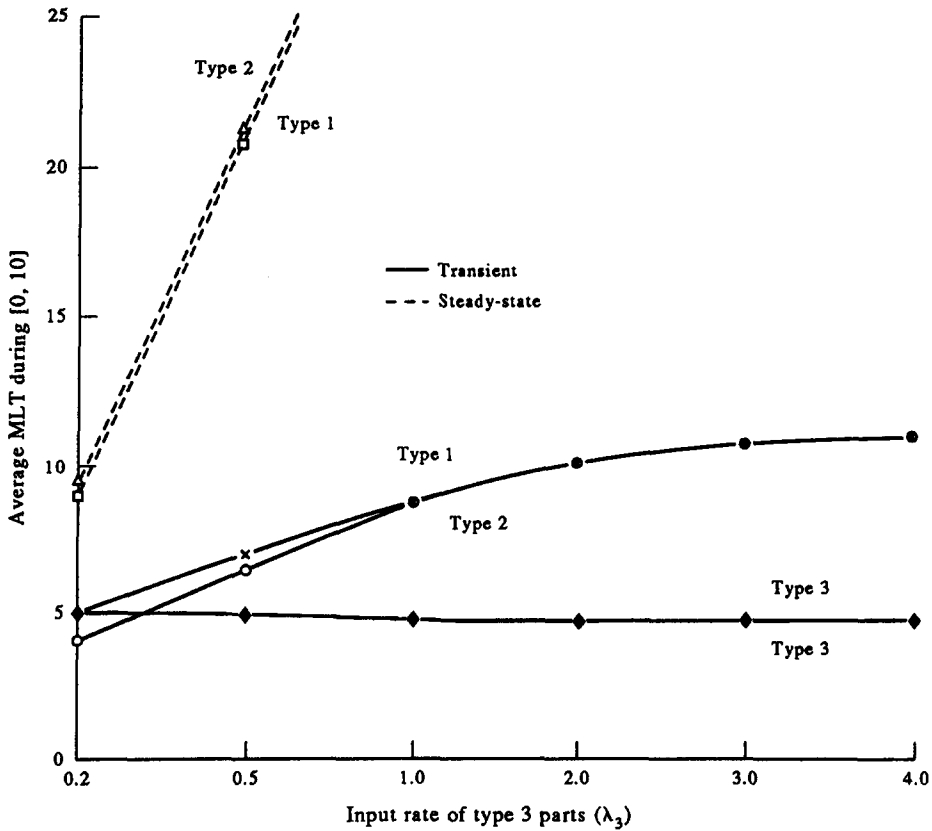


Fig. 4. Effect of arrival rate of SLP jobs on MLT values.

Specific time-dependent or steady-state performance measures can be obtained by substituting appropriate reward rates in individual states [11,4].

2.4. Categories of priority jobs

As already remarked, we consider two different classes of priority jobs: SLP (Slow-Long-Priority) jobs and FSP (Fast-Short-Priority) jobs. We consider the effect of these separately. SLP jobs have an arrival rate λ_3 which is much smaller than either λ_1 and λ_2 , but, the mean service time $1/\mu_3$ of each SLP job is much larger than both $1/\mu_1$ and $1/\mu_2$. Such jobs are quite common in real-world situations. We also assume that the setup times for SLP jobs are higher than those for type 1 and type 2 jobs.

FSP jobs are complementary to SLP jobs in the sense that they arrive at rapid intervals into the system, have low processing times and low machine setup times. That is, λ_3 is higher than either λ_1 and λ_2 ; μ_3 is higher than either μ_1 and μ_2 ; and s_3 is higher than either s_1 and s_2 . Such jobs are also not uncommon in manufacturing situations.

2.5. Experiments conducted

We carry out two separate sets of experiments. The first set is concerned with investigating the effect of SLP jobs and the second with the effect of FSP jobs. There are a large number of parameters involved with the model (observation period, input arrival rates, service rates, setup times, and buffer capacities) and potentially one can do a large number of experiments by varying different sets of parameters each time. Our interest in this paper is to study the effect of variations in two of these parameters: Observation period, t , and arrival rate of priority jobs, λ_3 . One can also study the effect of service rates, setup times, and buffer capacities.

3. EFFECT OF SLP JOBS

Here we consider type 3 jobs as constituting a slowly-arriving stream of jobs with large processing times. We assume $\lambda_1 = \lambda_2 = 4/h$; $\lambda_3 = 0.5/h$; $\mu_1 = 4/h$; $\mu_2 = 6/h$; $\mu_3 = 1.0/h$; $s_1 = 0.5/h$; $s_2 = s_3 = 0.4/h$ and $N_1 = N_2 = N_3 = 4$. In Section 3.3, we study the effect of λ_3 on the performance metrics, keeping all other

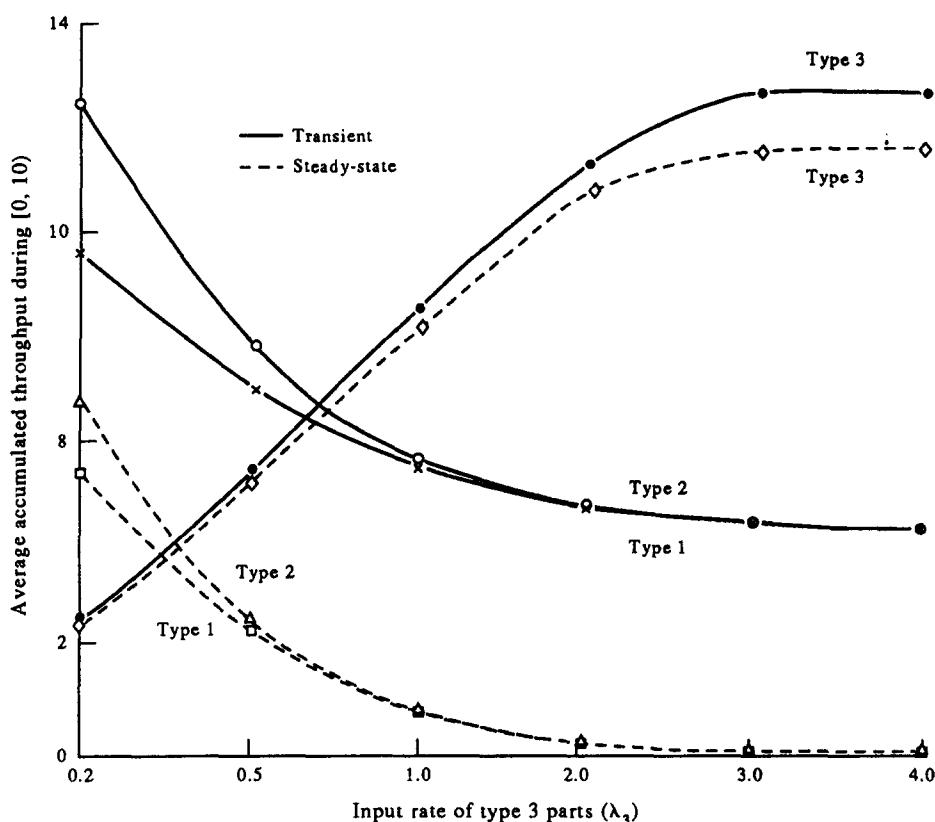


Fig. 5. Effect of arrival rate of SLP jobs on throughput values.

parameters constant at the above values.

Figures 2–5 give the results of the numerical experimentation we have done with SLP jobs. We report our findings here. In all the experiments, the initial state of the system is machine-idle with the machine set up for type 1 and all buffers empty.

Figures 2 and 3 give respectively, the average MLT and the average accumulated throughput during $[0, t]$ as t varies from 1 to 20 h. Figures 4 and 5 give respectively the average MLT during an interval of 10 h and the average accumulated throughput during an interval of 10 h, as a function of the arrival rate λ_3 of type 3 jobs, varying from 0.2 h to 4.0 h. In all these figures, performance estimates obtained by transient analysis are shown in solid lines while steady-state analysis estimates are shown in dotted lines.

3.1. Variation of MLT

From Fig. 2, we observe that the MLT values of type 3 jobs reach steady state in about 10 h, whereas those of nonpriority jobs don't reach steady state even at 20 h. As a result of the high priority accorded to type 3 jobs and their long processing times, the probability of the system processing type 3 for longer lengths of time is high and this explains the above observation. Also, MLT values of the parts are low as the system starts empty and hence the system is at light load in the initial phases. It is significant that steady-state values overestimate MLT of type 1 and type 2 parts. Thus, transient values are seen to be more credible estimates than steady-state values in the initial intervals of observations. We believe that this type of analysis helps in design and operation of a plant. For example, a plant manager can use this information to set due dates for low priority and high priority products.

3.2. Variation of accumulated throughput

Figure 3 gives the variation of accumulated throughput of the three types of jobs during interval $[0, t]$ as t varies from 1 to 18 h. For jobs of type 1 and type 2, note that there is a significant difference between transient values and steady-state values. In contrast, type 3 jobs virtually settle at steady-state production rate very early. As above, one can trace the cause to the high priority given to type 3 jobs and their large processing times. Also, steady-state values underestimate throughputs of the nonpriority jobs.

If the steady-state analysis were used for predicting the throughputs, then it would appear that the system is producing almost no jobs of type 1 and type 2, whereas, in fact it might be producing them with some throughput. The initial state has a major influence in determining these accumulated throughputs, and for an initial state different from what is considered here, one can expect different levels of throughput.

3.3. *Effect of arrival rate of high priority jobs*

In Figs 4 and 5, we let the arrival rate of the high priority class 3 jobs increase from 0.2/h to 4.0/h, i.e., slowly-arriving long jobs progressively become jobs with the same arrival rates as those of nonpriority job classes.

Figure 4 has the variation of the MLT over an interval [0,10] h as the input arrival rate of type 3 jobs varies from 0.2/h to 4.0/h. First, there is a large difference between steady-state MLT values and transient MLT values of type 1 and type 2 jobs. Also, this increases as λ_3 increases. Meanwhile, there is a perfect match between these two estimates in the case of type 3 jobs. In fact, the MLT of type 3 jobs slightly decreases as their arrival rate increases. One can explain these facts by noting that as λ_3 is increasing, the high priority jobs become more frequent, and hence the low priority jobs of type 1 and type 2 starve for service, i.e., the probability of the system switching over to type 1 or type 2 production becomes low. So with fewer switchovers and continued production of type 3 jobs, their MLT comes down. Hence, steady-state MLT values of type 1 and type 2 jobs are very high, as the initial conditions do not have any effect on the steady state.

Figure 5 depicts average accumulated throughput for [0,10] h interval versus arrival rate of type 3 jobs. There is a slight difference between the two estimates of type 3 jobs, whereas for type 1 and type 2 jobs the difference is pronounced. In fact, the steady-state accumulated throughput of nonpriority jobs virtually becomes zero at large arrival rate of type 3 jobs. As explained above, this is due to more and more priority jobs queuing up for service and wearing off the effect of the initial conditions on the performance of the system.

But what stands out is the fact that in the initial periods of evolution of the machine centre, even for

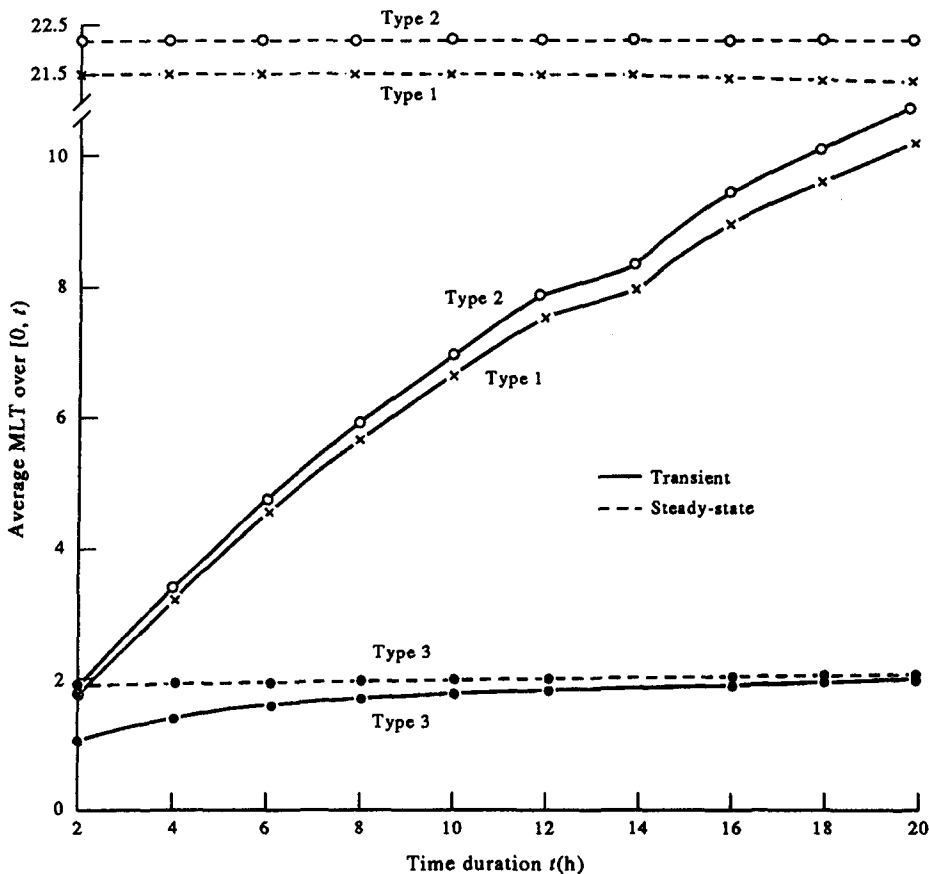


Fig. 6. Variation of MLT of FSP and nonpriority jobs.

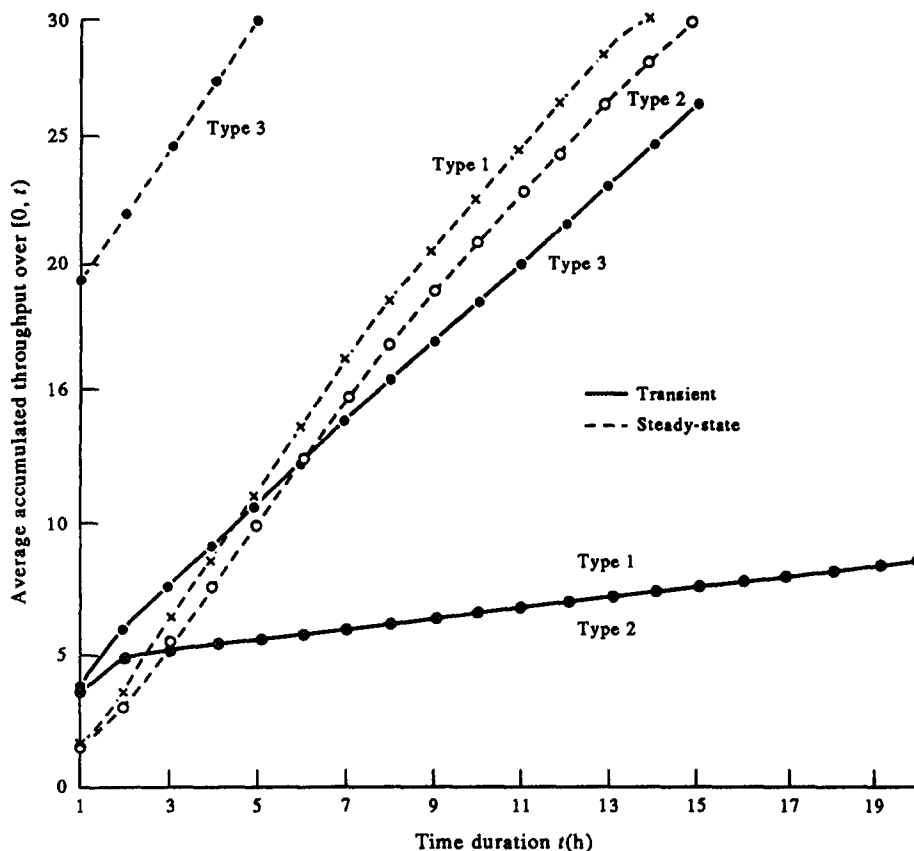


Fig. 7. Variation of throughput of FSP and nonpriority jobs.

large values of λ_3 , the accumulated throughput of the nonpriority jobs is not as low as indicated by steady-state estimates, nor are their MLT's as large as given by the steady-state estimates.

4. EFFECT OF FSP JOBS

In this section we look at the machine centre processing a class of priority jobs comprising a fast arriving stream of jobs with small processing times. We assume $\lambda_1 = \lambda_2 = 4/h$; $\lambda_3 = 20/h$; $\mu_1 = 4/h$; $\mu_2 = 6/h$; $\mu_3 = 30/h$; $s_1 = 0.5/h$; $s_2 = 0.4/h$; $s_3 = 5/h$; and $N_1 = N_2 = N_3 = 4$. As in Section 3.3, here also, we study the effect of λ_3 on the performance metrics, keeping all other parameters constant at the above values.

Figures 6–9 give the results of our study of the performance measures with FSP jobs. As above the initial condition is machine-idle with the machine setup for type 1, and no jobs in the system.

4.1. Variation of MLT

Figure 6 gives the variation of MLT of jobs for the interval $[0, t]$ h with t varying from 2 to 20 h. Type 3 jobs' MLT reaches steady state faster, while MLT of type 1 and type 2 jobs does not reach steady state quickly. Here type 3 jobs are of fast arrival rate with low processing times; hence there will be too many switchovers even in the initial period of operation and these switchover operations with their attendant setups contribute to a longer period for the settling of the system. This explains the comparatively slower settling of type 3 jobs' MLT to steady state than in the case of SLP jobs. It also explains the low throughput of all job types (see Fig. 7). Too fast arrival of type 3 jobs means that probability of nonpriority jobs getting serviced consecutively is low—the machine changes over to type 3 jobs virtually after serving every nonpriority job. This also explains the high MLT of type 1 and type 2 jobs here as compared to those of SLP jobs.

4.2. Variation of accumulated throughput

Figure 7 gives the accumulated throughput of jobs in the interval $[0, t]$ with t varying from 1 to 20 h. The transient accumulated throughput of jobs of type 1 and type 2 differ from steady-state values;

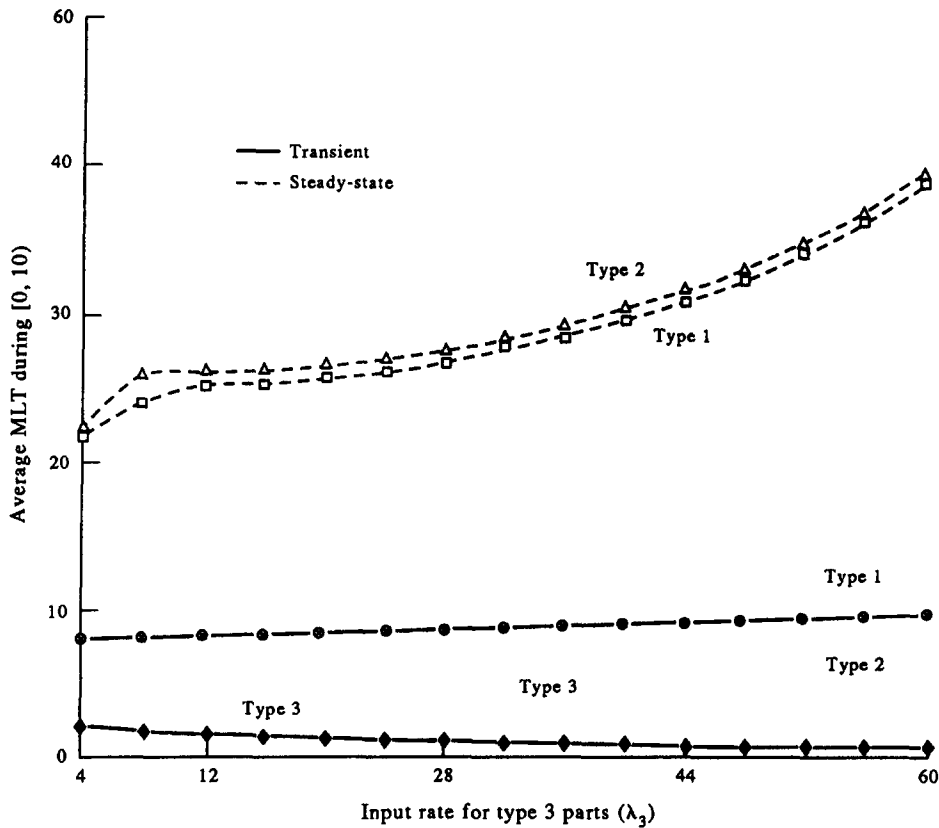


Fig. 8. Effect of arrival rate of FSP jobs on MLT values.

significantly, the steady-state accumulated throughput of type 3 also differs from transient values. As explained above, the frequent arrivals of type 3 jobs cause too many switchovers and the attendant setup operations consume much of the machine centre's operation period.

4.3. Effect of arrival rate of high priority jobs

In Fig. 8 we plot the average MLT of the jobs for the period [0,10] h as the arrival rate of type 3 increases from 4/h to 60/h. The steady-state and transient values of average MLT of type 3 jobs match for this entire range of arrival rates, but in the case of type 1 and type 2 jobs they differ. Again, the difference is not as drastic as in the case of SLP jobs. As λ_3 increases, more type 3 jobs are processed before a low-priority job is taken up for processing. Hence the MLT of low priority jobs increases and that of type 3 jobs decreases.

The final graph (Fig. 9) is a plot of average accumulated throughput during [0,10] h with respect to varying arrival rate of type 3 jobs (from 4/h to 40/h). It is clear that transient and steady-state values of accumulated throughput of type 1 and type 2 jobs are not very different from each other during a 10 h period of operation. The throughput of nonpriority jobs comes down as the rate λ_3 increases.

The corresponding values for type 3 differ slightly but progressively the difference increases. As the rate λ_3 increases, the possibility of nonpriority jobs being processed consecutively becomes less and hence, average accumulated throughput of these jobs becomes less.

5. CONCLUSIONS AND FUTURE WORK

The main objective of this article was to carry out transient analysis of a manufacturing system model, with a view to bringing out the importance of transient analysis and to show that transient analysis is more credible than steady-state analysis in many situations. By way of extensive numerical study of a manufacturing facility producing a set of routine jobs (nonpriority jobs here) and a set of priority jobs following priority scheduling and with significant setup times, we have studied the transient performance of a system in a more realistic setting than earlier studies.

The findings of this study are in two parts: the effect of the Slow-Long-Priority jobs and the effect of

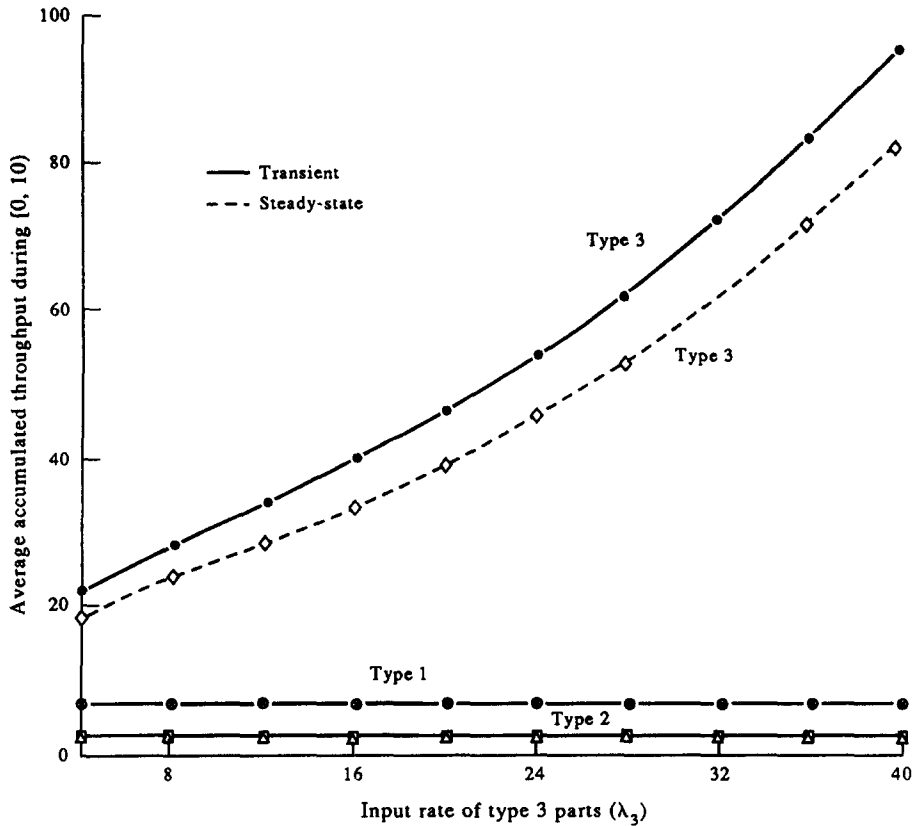


Fig. 9. Effect of arrival rate of FSP jobs on throughput values.

Fast-Short-Priority jobs. In the case of SLP jobs, average MLT and accumulated throughput settle at their steady-state values quickly. Also, steady-state analysis of nonpriority jobs overestimates the MLT and underestimates the throughput in the initial phase of system. As the arrival rate of priority jobs increases these effects become more pronounced. As for the FSP jobs, essentially the same observations hold, but the frequent switchovers entailed by the rapidly arriving priority jobs delay the approach to steady state. Consequently, transient analysis is even more important in this case. We have also shown that the transient analysis of the model has several important implications for the design of a system, such as in setting realistic due dates, estimating input arrival rates, and determining buffer capacities.

Some issues that need to be looked into include the effect of initial states, the simultaneous effect of SLP and FSP jobs, effect of different scheduling policies, etc. Also, we have only studied the effects of length of observation interval and the rate of arrivals of priority jobs on the transient performance measures. The effects of service times, setup times, and buffer capacities are also very important, but is beyond the scope of the present investigation. Moreover, these input variables can also have significant effects on the steady-state or long-term behaviour of the system. It will be worthwhile to look into these issues in the future.

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