An Optimal Mechanism for Sponsored Search Auction

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Introduction

- Problem Definition
Introduction

- Problem Definition: Sequence of Queries

[Diagram showing users (User 1, User 2, User N) sending queries (Q1, Q2, Q3) to Google].
Introduction

- Problem Definition: Bids, Valuations, and Click Probabilities

<table>
<thead>
<tr>
<th>Search Results</th>
<th>Sponsored Links</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_{i1}$</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\alpha_{i2}$</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>$\alpha_{im}$</td>
</tr>
</tbody>
</table>

Advertisers

- CPC
- Advertisers

b_1 $\theta_1$

b_2 $\theta_2$

b_n $\theta_n$
Introduction

- **Problem Definition: Bids, Valuations, and Click Probabilities**

\[ b = (b_1, \ldots, b_n) = \text{Bid vector of advertisers} \]
\[ b^{(1)}, \ldots, b^{(n)} = \text{Decreasing ordering of the bids} \]
\[ \theta_i = \text{Value derived out of a click by advertiser } i \]
\[ = \text{Type of advertiser } i \]
\[ \Theta_i = \text{Set of types of advertiser } i \]
\[ \theta = (\theta_1, \ldots, \theta_n) = \text{Type vector of advertisers} \]
\[ \alpha_{ij} = \text{Click probability of } i^{th} \text{ Ad in } j^{th} \text{ position} \]
\[ 1 \geq \alpha_{i1} \geq \alpha_{i2} \geq \cdots \geq \alpha_{im} \geq 0 \ \forall i \in N \ (\text{AAE Assumption}) \]
Introduction

- **Problem Definition: Search Engine’s Problem**

- **Allocation Rule**
  Who should be allocated what?
  \[ y_{ij}(b) = \begin{cases} 
  1 & \text{if advertiser } i \text{ is allocated slot } j \\
  0 & \text{o/w} 
  \end{cases} \]

- **Payment Rule**
  Which advertiser should be charged what price?
  \[ p_i(b) = \text{Price that is charged from advertiser } i \]
  for per click

- Google
  - \( b^{(1)} \)
  - \( b^{(2)} \)
  - \( b^{(m)} \)
Introduction

- **Recent Literature**


  - J. Feng, “Optimal Mechanism for selling a set of Commonly Ranked Objects”, *Mimeo, February 2005*


  - G. Aggarwal, A. Goel, and R. Motwani, “Truthful Auction for Pricing Search Keywords”, *ACM Conference on Electronic Commerce (EC’06), Ann Arbor, MI, June 11-15, 2006*

Outline

✓ Introduction
  ✓ Problem Definition
  ✓ Significance
  ✓ Recent Literature
  ▪ Three well known mechanisms
    ▪ Generalized First Price (GFP)
    ▪ Generalized Second Price (GSP)
    ▪ Vickrey-Clarke-Groves (VCG)
  ▪ A new mechanism – Optimal (OPT) Mechanism
  ▪ What is the best mechanism for Sponsored Search Auction?
  ▪ Comparison of OPT with GSP and VCG
    ▪ Incentive Compatibility
    ▪ Expected Revenue of the Search Engine
    ▪ Individual Rationality
    ▪ Computational Complexity
Generalized First Price (GFP)

\[ b_1 = 2 \]
\[ y_{11}(b) = 1 \]
\[ y_{12}(b) = 0 \]
\[ p_1(b) = 2 \]

\[ b_2 = 1.5 \]
\[ y_{21}(b) = 0 \]
\[ y_{22}(b) = 1 \]
\[ p_2(b) = 1.5 \]

\[ b_3 = 1 \]
\[ y_{31}(b) = 0 \]
\[ y_{32}(b) = 0 \]
\[ p_3(b) = 0 \]

\[ b = (2, 1.5, 1) \]
Generalized First Price (GFP)

- **Allocation Rule**
  Allocate the slots in decreasing order of bids

  $$y_{ij}(b) = \begin{cases} 
  1 & \text{if } b_i = b^{(j)} \text{ and } j \leq \min(m,n) \\
  0 & \text{o/w}
  \end{cases}$$

- **Payment Rule**
  For every user click, charge the advertiser his bid

  $$p_i(b) = \begin{cases} 
  b_i & \text{if advertiser } i\text{'s Ad is displayed} \\
  0 & \text{o/w}
  \end{cases}$$

*Introduced by Overture in 1997*
Generalized Second Price (GSP)

- **Allocation Rule**
  
  - **Yahoo Rule**
    Allocate the slots in decreasing order of bids
  
  - **Greedy Rule**
    Allocate 1st slot to advertiser $i_1 = \arg\max_{i \in N} (\alpha_i b_i)$
    Allocate 2nd slot to advertiser $i_2 = \arg\max_{i \in N \setminus i} (\alpha_i b_i)$

- **Google Rule**
  Allocate the slots in decreasing order of Ranking Score
  Ranking Score = $b_i \times CTR_i$

*Introduced by Google in 2002 (Above facts are based on literature)*
Generalized Second Price (GSP)

- **Payment Rule**
  - For every click, charge next highest bid + $0.01
  - The bottom most advertiser is charged highest disqualified bid +$0.01
  - charge 0 if no such bid
Generalized Second Price (GSP)

\[ b_1 = 2 \quad \text{with} \quad y_{11}(b) = 1, \quad y_{12}(b) = 0, \quad p_1(b) = 1.5 \]

\[ b_2 = 1.5 \quad \text{with} \quad y_{21}(b) = 0, \quad y_{22}(b) = 1, \quad p_2(b) = 1 \]

\[ b_3 = 1 \quad \text{with} \quad y_{31}(b) = 0, \quad y_{32}(b) = 0, \quad p_3(b) = 0 \]

\[ b = (2, 1.5, 1) \]
Generalized Second Price (GSP)

- Allocation Rule

Greedy

\[
\begin{bmatrix}
\alpha_{11} & \cdots & \alpha_{1m} \\
\vdots & \ddots & \vdots \\
\alpha_{n1} & \cdots & \alpha_{nm}
\end{bmatrix}
\begin{bmatrix}
b_1 \\
\vdots \\
b_n
\end{bmatrix}
\begin{bmatrix}
CTR_1 \\
\vdots \\
CTR_n
\end{bmatrix}
\]

Yahoo

Google

\[
CTR_i = \sum_{j=1}^{m} y_{ij} \alpha_{ij}
\]

\Rightarrow

\[
CTR_i \leq \sum_{j=1}^{m} \alpha_{ij}
\]
Generalized Second Price (GSP)

- **Learning CTR and Click Probabilities**
- **Average over Fixed Time Window**
  \[
  CTR_i = \frac{C_i}{I_i}; \quad \alpha_{ij} = \frac{C_{ij}}{I_{ij}}
  \]
- **Average over Fixed Impression Window**
  \[
  CTR_i = \frac{C_i}{1000}; \quad \alpha_{ij} = \frac{C_{ij}}{1000}
  \]
  \[I_i = 1000\]
- **Average over Fixed Click Window**
  \[
  CTR_i = \frac{100}{I_i}; \quad \alpha_{ij} = \frac{100}{I_{ij}}
  \]
  \[C_i = 100\]
Generalized Second Price (GSP)

- Relationship Among Allocation Rules

<table>
<thead>
<tr>
<th>(AE) Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
</tr>
</tbody>
</table>

\[
\sum_{i=1}^{n} b_i \left( \sum_{j=1}^{m} \alpha_{ij} y_{ij}(b) \right) = \sum_{i=1}^{n} b_i v_i(y(b))
\]

\[
\sum_{i=1}^{n} y_{ij}(b) \leq 1 \forall j \in M
\]

\[
\sum_{j=1}^{m} y_{ij}(b) \leq 1 \forall i \in N
\]

\[
0 \leq y_{ij} \forall i \in N, \forall j \in M
\]

- Proposition

Let click probabilities satisfy AAE assumption

- Greedy allocation rule is an optimal solution of the (AE) Problem

- If click probabilities depend only on identity of the advertiser and are independent of the position of the Ad, i.e. \( \alpha_{i1} = \alpha_{i2} = \cdots = \alpha_{im} = CTR_i \)
  then greedy rule and Google rule result in the same allocation

- If click probabilities depend only on position of the Ad and are independent of the identity of the advertiser, i.e. \( \alpha_{1j} = \alpha_{2j} = \cdots = \alpha_{nj} = \alpha_j \)
  then greedy rule and Yahoo! rule result in the same allocation
Vickrey-Clarke-Groves (VCG)

- **Allocation Rule**
  - Solution of (AE) Problem
  - Same as Yahoo! allocation under the assumption that click probability depends only on position

- **Payment Rule**

\[
t_i(b) = \left[ \sum_{j \neq i} b_j v_j(y^*_i(b)) \right] - \left[ \sum_{j \neq i} b_j v_j(y^*(b)) \right]
\]

\[
p^{(j)}(b) = \frac{t^{(j)}(b)}{\alpha_j}
\]
Vickrey-Clarke-Groves (VCG)

- **Payment Rule**

**Case 1** ($m < n$)

$$ p^{(j)}(b) = \begin{cases} 
\frac{1}{\alpha_j} \left[ \sum_{k=j}^{m-1} \beta_k b^{(k+1)} \right] + \frac{\alpha_m}{\alpha_j} b^{(m+1)} & \text{if } 1 \leq j \leq (m-1) \\
0 & \text{if } j = m \\
0 & \text{if } m < j \leq n 
\end{cases} $$

**Case 2** ($n \leq m$)

$$ p^{(j)}(b) = \begin{cases} 
\frac{1}{\alpha_j} \left[ \sum_{k=j}^{n-1} \beta_k b^{(k+1)} \right] & \text{if } 1 \leq j \leq (n-1) \\
0 & \text{if } j = n 
\end{cases} $$

where $\beta_k = (\alpha_k - \alpha_{k+1})$
Vickrey-Clarke-Groves (VCG)

\[ b_1 = 2.0 \]
\[ y_{11}(b) = 1 \]
\[ y_{12}(b) = 0 \]
\[ p_1(b) = 1.5 \left( 1 - \frac{\alpha_2}{3\alpha_1} \right) \]

\[ b_2 = 1.5 \]
\[ y_{21}(b) = 0 \]
\[ y_{22}(b) = 1 \]
\[ p_2(b) = 1 \]

\[ b_3 = 1.0 \]
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\[ p_3(b) = 0 \]
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✓ Introduction
  ✓ Problem Definition
  ✓ Significance
  ✓ Related Literature
✓ Three well known mechanisms
  ✓ Generalized First Price (GFP)
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  ✓ Vickrey-Clarke-Groves (VCG)
▪ A new mechanism – Optimal (OPT)
▪ What is the best mechanism for Sponsored Search Auction?
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  ▪ Computational Complexity
**Optimal (OPT)**

- **Allocation Rule**

  \[ y_{ij}(b) = \begin{cases} 
  0 & \forall 1 \leq j \leq n : \text{ if } J_i(b_i) < 0 \\
  1 & \forall 1 \leq j \leq m : \text{ if } J_i(b_i) = J^{(j)} \\
  0 & \forall m < j \leq n : \text{ if } J_i(b_i) = J^{(j)} 
\end{cases} \]

  where \( J^{(j)} \) is the \( j^{th} \) highest value among \( J_i(b_i) = \left( b_i - \frac{1 - \Phi_i(b_i)}{\phi_i(b_i)} \right) \)

  (Assumption: \( J_i(b_i) \) is non decreasing; True for Uniform, Exponential)

- **Proposition**

  - Advertisers are symmetric, i.e.
    \( \Theta_1 = \Theta_2 = \cdots = \Theta_n = \Theta \)
    \( \Phi_1(.) = \Phi_2(.) = \cdots = \Phi_n(.) \)
  - \( J_i(.) > 0 \forall i = 1, \cdots, n \)

  For a given bid vector \( b \), the OPT results in the same allocation as the GSP and the VCG, i.e. allocate in decreasing order of bids
Optimal (OPT)

- **Payment Rule**

**Case 1 (m < n)**

\[
p_i(b_i, b_{-i}) = \begin{cases} 
\frac{1}{\alpha_r} \left[ \sum_{k=r}^{m-1} \beta_k z_{ik}(b_{-i}) \right] + \frac{\alpha_m}{\alpha_r} z_{im}(b_{-i}) & \text{if } 1 \leq r \leq (m-1) \\
z_{im}(b_{-i}) & \text{if } r = m \\
0 & \text{o/w}
\end{cases}
\]

**Case 2 (n ≤ m)**

\[
p_i(b_i, b_{-i}) = \begin{cases} 
\frac{1}{\alpha_r} \left[ \sum_{k=r}^{n-1} \beta_k z_{ik}(b_{-i}) \right] + \frac{\alpha_n}{\alpha_r} z_{in}(b_{-i}) & \text{if } 1 \leq r \leq (n-1) \\
z_{in}(b_{-i}) & \text{if } r = n \\
0 & \text{o/w}
\end{cases}
\]

where

- **r** is the position at which advertiser \( j \) is allocated
- \( \beta_k = (\alpha_k - \alpha_{k+1}) \)
- \( z_{ij}(b_{-i}) \) is the minimum bid for the advertiser \( i \) which can make him win the \( j^{th} \) slot against the bid vector \( b_{-i} \) from other advertisers
Optimal (OPT)

- Payment Rule when Advertisers are Symmetric

\[ \Theta_1 = \Theta_2 = \cdots = \Theta_n = \Theta = [L,U] \]
\[ \Phi_1(.) = \Phi_2(.) = \cdots = \Phi_n(.) \]

**Case 1** \((m < n)\)

\[
p_i(b_i, b_{-i}) = \begin{cases} 
\frac{1}{\alpha_r} \left[ \sum_{k=r}^{m-1} \beta_k b^{(k+1)} \right] + \frac{\alpha_m}{\alpha_r} b^{(m+1)} & \text{if } 1 \leq j \leq (m-1) \\
0 & \text{if } j = m \\
0 & \text{if } m < j \leq n 
\end{cases}
\]

**Case 2** \((n \leq m)\)

\[
p_i(b_i, b_{-i}) = \begin{cases} 
\frac{1}{\alpha_r} \left[ \sum_{k=r}^{n-1} \beta_k b^{(k+1)} \right] + \frac{\alpha_n}{\alpha_r} L & \text{if } 1 \leq j \leq (n-1) \\
L & \text{if } j = n 
\end{cases}
\]
**Proposition**

- Advertisers are symmetric, i.e.
  \[ \Theta_1 = \Theta_2 = \cdots = \Theta_n = \Theta = [L, U] \]
  \[ \Phi_1(.) = \Phi_2(.) = \cdots = \Phi_n(.) \]
  \[ J_i(.) > 0 \ \forall i = 1, \cdots, n \]
  \[ m < n \]

- Advertisers are symmetric, i.e.
  \[ \Theta_1 = \Theta_2 = \cdots = \Theta_n = \Theta \]
  \[ \Phi_1(.) = \Phi_2(.) = \cdots = \Phi_n(.) \]
  \[ J_i(.) > 0 \ \forall i = 1, \cdots, n \]
  \[ m = n \]

---

**Payment Rule**

\[ \text{OPT} \equiv \text{VCG} \]

(up to a constant factor L)
Example: OPT

\( \Theta_1 = [1,2] \)
\( \Phi_1(x) = (x - 1); \phi_1(x) = 1 \)
\( J_1(2) = 2 - \frac{1-1}{1} = 2 \)
\( y_{11}(b) = 1; p_1(b) = 1.5 \left( 1 - \frac{\alpha_2}{3 \alpha_1} \right) \)

\( \Theta_2 = [1,2] \)
\( \Phi_2(x) = (x - 1); \phi_2(x) = 1 \)
\( J_2(1.5) = 1.5 - \frac{1-0.5}{1} = 1 \)
\( y_{22}(b) = 1; p_2(b) = 1 \)

\( \Theta_3 = [1,2] \)
\( \Phi_3(x) = (x - 1); \phi_3(x) = 1 \)
\( J_3(1) = 1 - \frac{1-0}{1} = 0 \)
\( y_{3j} = 0; p_3(b) = 0 \)
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What is the best Mechanism for Sponsored Search Auction?

- **Search Engine’s View Points**

  - Economic and Computational Performance measures
  - The advertisers’ equilibrium bidding strategy profile \((s_1^*(.), \ldots, s_n^*(.))\)
  - Effect of \((s_1^*(.), \ldots, s_n^*(.))\) on performance measures
What is the best Mechanism for Sponsored Search Auction?

- **Economic and Computational Performance Measures**
  - Revenue Maximization
  - Individual Rationality (IR)
  - Incentive Compatibility (IC)
  - Computational Complexity
What is the best Mechanism for Sponsored Search Auction?

- Sponsored Search Auction as a Mechanism Design Problem

\[ f(b) = (y_{ij}(b), p_i(b))_{i \in N, j \in M} \]

(Allocation Rule, Payment Rule)

\[ u_i(f(b), \theta_i) = v_i(y(b)) (\theta_i - p_i(b)) = \left( \sum_{j=1}^{m} \alpha_j y_{ij}(b) \right) (\theta_i - p_i(b)) \]
What is the best Mechanism for Sponsored Search Auction?

- **Strategic Bidding Behavior of Advertisers**
  If all the advertisers are rational and intelligent and this fact is common knowledge then each advertiser’s expected bidding behavior is given by

- **Dominant Strategy Equilibrium (DSE)**
  Strategy profile \((s^*_1(.), \ldots, s^*_n(.))\) is said to be dominant Strategy equilibrium iff
  \[
  u_i(f(s^*_i(\theta_i), b_{-i})), \theta_i) \geq u_i(f(b_i, b_{-i})), \theta_i) \forall b_i \in \Theta_i, \forall b_{-i} \in \Theta_{-i}
  \]

- **Bayesian Nash Equilibrium (BNE)**
  Strategy profile \((s^*_1(.), \ldots, s^*_n(.))\) is said to be Bayesian Nash equilibrium iff
  \[
  E_{\theta_i} \left[u_i(f(s^*_i(\theta_i), s^*_{-i}(\theta_{-i})), \theta_i) \mid \theta_i \right] \geq E_{\theta_{-i}} \left[u_i(f(b_i, s^*_{-i}(\theta_{-i})), \theta_i) \mid \theta_i \right] \forall b_i \in \Theta_i
  \]
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✓ What is the best mechanism for Sponsored Search Auction?
  ▪ Comparison of OPT with GSP and VCG
    ▪ Incentive Compatibility
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    ▪ Individual Rationality
    ▪ Computational Complexity
Comparison of OPT with GSP and VCG

- Incentive Compatibility

  - **VCG:** Follow $s_i^*(\theta_i) = \theta_i$ irrespective of what the others are doing (DSE)

  - **OPT:** Follow $s_i(\theta) = \theta$ if all rivals are also doing so (BNE)

  - **GSP:** Never follow strategy $s_i^*(\theta_i) = \theta_i$. Use the following BNE strategy

$$s_i^*(\theta_i) = \begin{cases} 
\theta_i - \frac{1}{g(\theta_i, (m-1))} \int_{\theta_i}^{\theta_i} f(x, \theta_i, (m-1))s'(x)dx : \text{if } n = m \\
\theta_i - \frac{1}{g(\theta_i, m)} \int_{\theta_i}^{\theta_i} f(x, \theta_i, m)s'(x)dx : \text{if } m < n 
\end{cases}$$

$$f(x, \theta_i, k) = \sum_{j=1}^{k} (j-1)\alpha_j^{n-1}C_{j-1}(\overline{\Phi}(\theta_i))^{j-2}(\Phi(\theta_i))^{n-j}$$

$$g(\theta_i, k) = k\alpha_k^{n-1}C_k(\overline{\Phi}(\theta_i))^{k-1}(\Phi(\theta_i))^{n-k-1} + \sum_{j=1}^{k-1} j(\alpha_j - \alpha_{j+1})^{n-1}C_j(\overline{\Phi}(\theta_i))^{j-1}(\Phi(\theta_i))^{n-j-1}$$
Comparison of OPT with GSP and VCG

- Expected Revenue Earned by the Search Engine

Revenue Equivalence Theorem:

Consider a sponsored search auction setting, in which

1. The advertisers are risk neutral
2. The advertisers are symmetric
3. For each advertiser $i$, we have $\phi_i(.) > 0$
4. The advertisers draw their types independently

Consider two different mechanisms, each having symmetric and increasing Bayesian Nash equilibrium such that

1. For each possible $(\theta_1, \ldots, \theta_n)$ the final allocation is the same
2. Each advertiser $i$ has same expected utility in two mechanisms for $\theta_i = L$

then equilibria of two mechanisms generate the same expected revenue for the search engine
Comparison of OPT with GSP and VCG

- **Expected Revenue Earned by the Search Engine**

- Revenue Equivalence of GSP, VCG, and OPT Mechanisms

  Consider a sponsored search auction setting, in which
  
  1. The advertisers are risk neutral
  2. The advertisers are symmetric
  3. For each advertiser $i$, we have $\phi_i(.) > 0$
  4. The advertisers draw their types independently
  5. For each advertiser $i$, we have $J_i(.) > 0$ and $J_i(.)$ is non-decreasing

  Consider three different auction mechanisms – GSP, VCG, and OPT. Let $R_{GSP}$, $R_{VCG}$ and $R_{OPT}$ be the expected revenue earned by the search engine under these three mechanisms against every query received, then

  $R_{GSP} = R_{VCG} = R_{OPT}$ if $m < n$
  
  $R_{VCG} \leq R_{GSP} \leq R_{OPT}$ if $n \leq m$
Comparison of OPT with GSP and VCG

- Expected Revenue of Search Engine

**Case 1** \((m < n)\)

\[
R_{OPT} = n \left[ \int_{L}^{U} m \alpha_m^{n-1} C_m(\Phi(x))^m(\Phi(x))^{n-m-1} + \sum_{j=1}^{m-1} j \beta_j^{n-1} C_j(\Phi(x))^j(\Phi(x))^{n-j-1} \right] x\phi(x) dx
\]

**Case 2** \((n \leq m)\)

\[
R_{OPT} = n \left[ \alpha_n L + \int_{L}^{U} \left( \sum_{j=1}^{m-1} j \beta_j^{n-1} C_j(\Phi(x))^j(\Phi(x))^{n-j-1} \right) x\phi(x) dx \right]
\]

\[
R_{VCG} = n \left[ \int_{L}^{U} \left( \sum_{j=1}^{m-1} j \beta_j^{n-1} C_j(\Phi(x))^j(\Phi(x))^{n-j-1} \right) x\phi(x) dx \right]
\]
Comparison of OPT with GSP and VCG

- Economic Performance of Auction Mechanisms

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<th>BIC</th>
<th>IR</th>
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<td>Decreasing order of</td>
<td>Next Highest bid</td>
<td>X</td>
<td>X</td>
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</tr>
<tr>
<td>VCG</td>
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<td>✔</td>
<td>✔</td>
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Comparison of OPT with GSP and VCG

- Economic Performance of Auction Mechanisms

- Allocative Efficient
- Individually Rational
- Dominant Strategy
- Incentive Compatible
- Bayesian Incentive Compatible

- GFP
- GSP
- VCG
- OPT
Comparison of OPT with GSP and VCG

- Experimental Results

![Graph showing comparison of OPT, VCG, and GSP with expected revenue on the y-axis and number of advertisers on the x-axis. Parameters: No. of Slots (m) = 10, Type Interval (θ) = [1, 3], Click Prob. Distribution = Pseudo-Geometric, α = 0.5, r = 0.5]
Comparison of OPT with GSP and VCG

- Experimental Results

![Graph showing comparison of OPT with GSP and VCG]

- No. of Slots (m) = 10
- Type Interval ($\Theta$) = [1, 3]
- Click Prob. Distribution = Pseudo-Geometric
- $\alpha_1 = 0.5$
- $r = 0.5$
Comparison of OPT with GSP and VCG

- Experimental Results

![Graph showing comparison of expected revenue with number of advertisers](image)

- Number of Slots (m) = 15
- Type Interval $(\Theta)=[1,3]$  
- Click Prob. Distribution = Pseudo-Geometric  
  - $\alpha_1 = 0.5$  
  - $r = 0.5$
Comparison of OPT with GSP and VCG

- **Experimental Results**

![Graph showing comparison of OPT, GSP, and VCG]

- No. of Slots (m) = 15
- Type Interval (Θ) = [1, 3]
- Click Prob. Distribution = Pseudo-Geometric
- α₁ = 0.5
- r = 0.5
Comparison of OPT with GSP and VCG

- Computational Performance of Auction Mechanisms

<table>
<thead>
<tr>
<th></th>
<th>Computational Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSP</td>
<td>(O(n \log n))</td>
</tr>
<tr>
<td>VCG</td>
<td>(O(n \log n + (\min(m, n))^2))</td>
</tr>
<tr>
<td>OPT</td>
<td>(O(n \log n + (\min(m, n))^2))</td>
</tr>
</tbody>
</table>
Introduction

Problem Definition
Significance
Related Literature

Three well known mechanisms
Generalized First Price (GFP)
Generalized Second Price (GSP)
Vickrey-Clarke-Groves (VCG)

A new mechanism – Optimal (OPT)

What is the best mechanism for Sponsored Search Auction?

Comparison of OPT with GSP and VCG
Incentive Compatibility
Expected Revenue of Search Engine
Individual Rationality
Computational Complexity
Future Directions

- Long Term Goals versus Short Term Goals
- Daily Budget
- Learning the Valuation Distribution $\Phi_i(.)$
- Assumption of Independence of Click Probability on Advertisers’ Identity
- Revenue Performance under Asymmetric Advertisers
- Click Fraud
- Competing Search Engines
- Optimal Bidding Strategy of the Advertisers
Questions and Answers …

Thank You …