# Tutorial: Game Theoretic Models for Social Network Analysis

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## ABSTRACT

The existing methods and techniques for social network analysis are inadequate to capture both the behavior (such as rationality and intelligence) of individuals and the strategic interactions that occur among these individuals. Game theory is a natural tool to overcome this inadequacy since it provides rigorous mathematical models of strategic interaction among autonomous, intelligent, and rational agents. Motivated by the above observation, this tutorial provides the conceptual underpinnings of the use of game theoretic models in social network analysis. In the first part of the tutorial, we provide rigorous foundations of relevant concepts in game theory and social network analysis. In the second part of the tutorial, we present a comprehensive study of four contemporary and pertinent problems in social networks: social network formation, determining in influential individuals for viral marketing, query incentive networks, and community detection.

## Categories and Subject Descriptors

F.2.2 [**Analysis of Algorithms and Problem Complexity**]: Nonnumerical Algorithms and Problems

## General Terms

Economics, Theory

## Keywords

Social networks, game theory, viral marketing, network formation, query incentive networks, community detection

## 1. INTRODUCTION

Social network analysis (SNA) [19] comprises a well developed suite of measures and metrics based on techniques such as graph theory, spectral theory, optimization theory. All this machinery in SNA is useful to measure the structural and statistical properties of social networks. In fact, generative models can reproduce networks with similar/same structural properties. However, the current SNA approaches are inadequate for the following reasons:

1. They do not satisfactorily capture the behavior of the individuals in social networks. For example, very often, the individuals in the network exhibit strategic behavior.

2. They do not explicitly capture the dynamics of strategic interaction among the individual in the networks.

Game theory [12, 13] is a natural tool to overcome these fundamental problems, as it provides a rich suite of mathematical models of strategic interaction among autonomous, intelligent, and rational individuals (or players). Game theoretic models are thus best suited to capture the strategic nature of individuals making up a social network. Game theoretic models are also complementary to the current SNA approaches and hence they add a new dimension to the area of SNA. Recently there have been several efforts in following a game theoretic approach to social network modeling [18, 7, 5, 10, 15, 6, 11, 9, 2, 1, 17, 16].

In our view, game theoretic models are appropriate for social network analysis from two perspectives:
(a) Game theoretic models are very natural for several problems in social network analysis. A few such problems include social network formation [7, 5, 10, 15], social network monetization [6], and bargaining on networks [11].
(b) Game theoretic models are useful as a tool to solve certain interesting problems in social network analysis. This leads to not only a deeper understanding of those problems but also efficient algorithms. A few examples of this kind include query incentive networks [9, 2], discovering communities in networks [1, 17], determining influential individuals for viral marketing [16], etc.

## 2. CONTENT OF THE TUTORIAL

Here we present a brief description of the material and the results that we discuss in this tutorial.

### 2.1 Social Network Analysis: A Quick Primer

First, we present a quick primer on social network analysis from Easley and Kleinberg [4]. We define important metrics for social network analysis, prominent approaches for social network analysis, and list a few benchmark problems.

### 2.2 Foundational Concepts in Game Theory

Here we first present the basic concepts from both non-cooperative game theory and cooperative game theory. Majority of this content is covered from Myerson [12]. We then present the foundational principles of mechanism design theory from Narahari, Garg, Ramasuri, and Hastagiri [13].


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## 2.3 Social Network Formation

The behavior of networks is driven by the actions of a large number of autonomous individuals, each motivated by self-interest and individual objectives. As a consequence of this, the global performance of such networks, which are the equilibrium outcomes of decentralized strategic interactions, can be worse than that of a network that is enforced by a central authority. In the literature, networks that are enforced by a central authority are known as efficient networks. Understanding the compatibility between the equilibrium networks and efficient networks is the primary focus of research in network formation. Most of the results that we cover in this context are primarily from Jackson [7], Goyal [5], Kleinberg, Suri, Tardos, and Wexler [10], and Ramasuri and Narahari [15].

## 2.4 Discovering Influential Individuals for Viral Marketing

Viral Marketing is based on the conceptual framework of diffusion of information. In this context, given a integer value $k$, it is very challenging to determine top-$k$ influential individuals to maximize the volume of information cascade. Formally, we define an objective function $\sigma(.)$ as follows. We note that an individual is active if he/she adopts the product or technology. If $S$ is the set of initially active nodes, then $\sigma(S)$ is the expected number of active nodes at the end of the diffusion process. For a given constant $k$, the top-$k$ nodes problem seeks to find a subset of nodes $S$ of cardinality $k$ that maximizes the value of $\sigma(S)$. In this setting, we first cover certain fundamental results from Domingos and Richardson [3], Kempe, Kleinberg, and Tardos [8]. Then we present a game theoretic approach to address the top-$k$ nodes problem and we present a few results from Ramasuri and Narahari [16].

## 2.5 Query Incentive Networks

We consider scenarios where a person in a social network is seeking some information from the social network and the other connected individuals forward the query down in the network as well as report back the answer, if any. Since every individual is an intelligent and rational agent and since forwarding the query (and then reporting back the answer) requires a certain amount of effort on her part, she may not be willing to do so. While offering an appropriate incentive to the intermediate nodes will increase the total reward which must be offered by the person posing the query, it will also increase the exposure of the query. This concept was captured by the model of Raghavan and Kleinberg [14] who called these *query incentive networks*. In the tutorial, we will bring out the role of game theory and mechanism design in the design of incentives in such networks [14, 2].

## 2.6 Community Detection in Social Networks

A community structure in a network is a division of network nodes into groups within which the network connections are dense, but between which the connections are sparse. Detecting communities in networks helps to understand the underlying characteristics of large networks. We first mention several variants of community detection in networks such as graph partitioning, finding the most dense subgraph in a given graph. Next we briefly mention various techniques for community detection like spectral methods, multi-level methods, optimization methods, and methods based on social network analysis. We then motivate the need for a game theoretic approach to community detection and present two game theoretic models for community detection Chen, Liu, Sun, and Wang [1] and Ramasuri and Narahari [17].

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