Motivation

- **Manipulation**: voters may get better outcome by misreporting their votes.
- Every reasonable voting rule is manipulable [4, 6].
- Domain restriction is a proposed solution: there exist nonmanipulable voting rules in restricted domains. However, the social planner often not sure about the domain.
- Computational intractability: manipulation is intractable for many commonly used voting rules [1, 2]. However, every reasonable voting rule is easy to manipulate in the average case [5].
- No satisfactory solution for preventing manipulation till today.
- Manipulation detection in real life:

**Automatic Detection of Manipulation is needed**

Preliminaries

- \( V \) - a set of \( n \) voters.
- \( C \) - a set of \( m \) candidates.
- Vote - a complete order over \( C \).
- \( L(C) \) - set of complete orders over \( C \).
- Voting rule - \( r : L(C) \rightarrow C \).
- Scoring rule - defined by \( \bar{a} = (\alpha_1, \ldots, \alpha_m) \in \mathbb{R}^m \).
- Bucklin rule - winner is the candidate getting majority within minimum number of top positions.
- Maximin rule - winner is the candidate with minimum margin of victory in its worst pairwise election.

Problem Formulation

**Coalition of possible manipulators**: Given an \( r \)-election, a subset of voters \( M \subset V \) is a CPM if there exists \( y \) such that:

\[
r((\preceq_M \setminus y) \cup V, M) >_M n \setminus x \setminus y \setminus M
\]

We call \( r((\preceq_M \setminus y) \cup V, M) \) the actual winner.

**Input**: election

**Find**: a coalition of possible manipulators \( M \) with \( |M| = k \).

Results

**Summary of Results**

<table>
<thead>
<tr>
<th>Voting Rule</th>
<th>CPM, ( k = 1 )</th>
<th>CPMW, ( k = 1 )</th>
<th>( k ) denotes coalition size</th>
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<td><strong>Scoring Rules</strong></td>
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<td>Borda</td>
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<td>Bucklin</td>
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<td>( P )</td>
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Borda: manipulation is NPC [3].

Borda: Detecting manipulation is easy.

Scoring Rules

**Theorem 1**: For scoring rules with \( \alpha_1 - \alpha_2 \leq \alpha_i - \alpha_{i+1}, \forall i \), the CPMW and the CPM problems are in \( P \), for any coalition size.

**Proof sketch**:
- Enough to prove for the CPMW problem in this setting.
- Let \( x \) be the current winner and \( y \) be the given actual winner. Let \( M \) be the subset of voters. Let \((r((\setminus x \cup y) \cup V, M))\) be the reported preference profile.
- Without loss of generality, we assume that \( x \) is the most preferred candidate in every \( y \), \( i \in M \).
- Let us define \( y \), \( i \in M \), by moving \( y \) to the second position in the preference of \( y \).
- In the profile \((r((\setminus x \cup y) \cup V, M))\), the winner is either \( x \) or \( y \) since only \( y \)'s score has increased.
- We claim that \( M \) is a coalition of possible manipulators with respect to \( y \) if and only if \( y \) is the winner in preference profile \((r((\setminus x \cup y) \cup V, M))\).

**Corollary 1**: For the Borda voting rule, the CPM and the CPM problems are in \( P \), for any coalition size.

**Bucklin Voting Rule**

**Lemma 1**: Consider a preference profile \((\preceq_M)\), where \( x \) is the winner with respect to the Bucklin voting rule. Suppose a subset of voters \( M \subset V \) form a coalition of possible manipulators. Let \( y \) be the actual winner. Then there exist preferences \((\preceq_M)\)'s such that \( y \) is a Bucklin winner in \((r((\setminus x \cup y) \cup V, M))\), and further:

1. \( y \) immediately follows \( x \) in each \( \preceq_M \).
2. The rank of \( x \) in each \( \preceq_M \) is in one of the following - first, \( b(y) - 1 \), \( b(y) \), \( b(y) + 1 \), where \( b(y) \) be the Bucklin score of \( y \) in \((r((\setminus x \cup y) \cup V, M))\).

**Theorem 1**: The CPMW problem and the CPM problems for Bucklin voting rule are in \( P \) for any coalition size.

**Proof sketch**:
- Enough to prove for the CPMW problem in this setting.
- Let \( x \) be the current winner and \( y \) be the given actual winner.
- For any final Bucklin score \( b(y) \) of \( y \), there are polynomially many possibilities for the positions of \( x \) and \( y \) in the profile of \( y \), \( i \in M \), since Bucklin voting rule is anonymous.
- Once the positions of \( x \) and \( y \) are fixed, we try to fill the top \( b(y) \) positions of each \( \preceq_M \), place a candidate in an empty position above \( b(y) \) in any \( \preceq_M \), if doing so does not make \( y \) lose the election.
- If we are able to successfully fill the top \( b(y) \) positions of all \( \preceq_M \), for all \( i \in M \), then \( M \) is a coalition of possible manipulators.
- If the above process fails for all possible above mentioned positions of \( x \) and \( y \) and all possible guesses of \( b(y) \), then \( M \) is not a coalition of possible manipulators.

Conclusion and Future work

- In this work, we have initiated a promising research direction for detecting manipulation in elections.
- Certainly there will be false positive outputs of our algorithms. Verifying the number of false manipulators that this model catches in a real or synthetic data set, where, we already have some knowledge about the manipulators, would be interesting.

References