Detecting Possible Manipulators in Elections

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Indian Institute of Science Bangalore

May 8, 2015

### Motivation

### Voting

### Applications of Voting

Top 250 movies as voted by our users

For this top 250, only votes from regular users are considered.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Rating</th>
<th>Title</th>
<th>Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.5</td>
<td>The Dark Knight (2008)</td>
<td>230,275</td>
</tr>
<tr>
<td>2</td>
<td>9.1</td>
<td>The Godfather (1972)</td>
<td>266,072</td>
</tr>
<tr>
<td>3</td>
<td>9.1</td>
<td>The Shawshank Redemption (1994)</td>
<td>229,592</td>
</tr>
<tr>
<td>5</td>
<td>8.9</td>
<td>Buono, Ill Vullo, Il poliziotto (1980)</td>
<td>66,661</td>
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<tr>
<td>6</td>
<td>8.9</td>
<td>Pulp Fiction (1994)</td>
<td>291,734</td>
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<tr>
<td>7</td>
<td>8.8</td>
<td>Schindler's List (1993)</td>
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<td>8</td>
<td>8.8</td>
<td>One Flew Over the Cuckoo's Nest (1975)</td>
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<tr>
<td>9</td>
<td>8.8</td>
<td>Star Wars: Episode V - The Empire Strikes Back (1980)</td>
<td>204,688</td>
</tr>
<tr>
<td>10</td>
<td>8.8</td>
<td>Casablanca (1942)</td>
<td>125,688</td>
</tr>
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Top 250 movies as voted by our users

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Applications of Voting

Top 250 movies as voted by our users

For this top 350, only votes from regular users are considered.

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<td>9.1</td>
<td>The Shawshank Redemption (1994)</td>
<td>208,555</td>
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<td>5</td>
<td>9.0</td>
<td>Buono Primo, Il secondo (1980)</td>
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<td>7</td>
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<td>Scent of a Woman (1992)</td>
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<td>8</td>
<td>8.8</td>
<td>One Flew Over the Cuckoo’s Nest (1975)</td>
<td>148,944</td>
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<td>9</td>
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<td>Star Wars: Episode V - The Empire Strikes Back (1980)</td>
<td>204,088</td>
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<td>10</td>
<td>8.8</td>
<td>Schindler’s List (1993)</td>
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<td>4</td>
<td>9.0</td>
<td>The Godfather: Part I (1972)</td>
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<td>9.0</td>
<td>Buono, il bello è l'inizio (II) (1960)</td>
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<td>Pulp Fiction (1994)</td>
<td>261,728</td>
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<td>Star Wars: Episode II - The Phantom Menace (1999)</td>
<td>204,888</td>
</tr>
<tr>
<td>10</td>
<td>8.8</td>
<td>For Whom The Bell Tolls (1942)</td>
<td>125,968</td>
</tr>
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Wikipedia

The Free Encyclopedia

Google

Yahoo!
Motivation

Manipulation in Election

- Misreporting votes may lead to better outcome
- "Reasonable" voting rules are manipulable

[Google, Yahoo! Images]
Motivation

Manipulation in Election

Motivation

Manipulation: misreporting votes may lead to better outcome. "Reasonable" voting rules are manipulable [Gibbard, 1973; Satterthwaite, 1975].

Dept of CSA (IISc)  Manipulation Detection in Voting  May 8, 2015
Motivation

Manipulation: misreporting votes may lead to better outcome
**Motivation**

Manipulation: misreporting votes may lead to better outcome

‘Reasonable’ voting rules are manipulable [Gibbard, 1973; Satterthwaite, 1975]
Motivation

Have non-manipulable rules in restricted domains

😊
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Computational intractability
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Not sure about domain 😞

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Computational intractability
Easy on average 😞
Motivation

Have non-manipulable rules in restricted domains
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Computational intractability
Easy on average 😞

No satisfactory solution for manipulation prevention
Manipulation detection in real life:

Figure: London Olympic 2012

Figure: Fifa World Cup 1982
Motivation

Manipulation detection in real life:

Figure: London Olympic 2012

Figure: Fifa World Cup 1982

Main Contribution

Formal study of manipulation detection
Voting Setting

Setting

- A set $C$ of $m$ candidates
- A set $V$ of $n$ votes
- Vote - a complete order over $C$
- Voting rule - $r : \mathcal{L}(C)^n \rightarrow C$

Example

- $C = \{x, y, z\}$
- Votes
  - Vote 1: $x > y > z$
  - Vote 2: $z > y > x$
  - Vote 3: $x > z > y$

Plurality rule: winner is candidate with most top positions

Plurality winner: $x$
Voting Setting

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  - Plurality winner: $x$
Example: Scoring Rules

Scoring Rule

- Score vector: \((\alpha_1, \ldots, \alpha_m) \in \mathbb{R}^m\)
- A vote \(x_1 > x_2 > \cdots > x_m \Rightarrow x_i\) gets score \(\alpha_i\)
- Winner: candidate with highest score

Important Special Cases

- Plurality: \((1, 0, \cdots, 0)\)
- Veto: \((0, \cdots, 0, -1)\)
- Borda: \((m - 1, m - 2, \cdots, 0)\)

Figure: Borda rule
Problem Definition

Problem Formulation

Fix any voting rule $r$

Example

$r(\cdots, \succ, \cdots) = x$

reported preference of voter $i$
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Problem Formulation

Fix any voting rule $r$

Example

$r(\ldots, \succ, \ldots) = x$

reported preference of voter $i$

$r(\ldots, \ldots \succ x \succ \ldots \succ y \succ \ldots, \ldots) = y$

possible actual preference of voter $i$
Problem Definition

Problem Formulation

Fix any voting rule $r$

Example

$$r(\ldots, >, \ldots) = x$$

reported preference of voter $i$

$$r(\ldots, \ldots \succ x \succ \ldots \succ y \succ \ldots, \ldots) = y$$

possible actual preference of voter $i$

Coalition of possible manipulators

A subset of voters $M \subset V$ is a CPM if there exists $\succ'_M$ such that:

$$r(\succ_M, \succ_{V \setminus M}) \succ'_M r(\succ'_M, \succ_{V \setminus M})$$

We call $r(\succ'_M, \succ_{V \setminus M})$ the actual winner
Problem Formulation

Fix any voting rule $r$

Input: election
Fix any voting rule \( r \)

Input: election

\( M \subset V \) given. Question: is \( M \) a coalition of possible manipulators?
Problem Definition

Problem Formulation

Fix any voting rule \( r \)

Input: election

\( M \subset V \) given. Question: is \( M \) a coalition of possible manipulators?

Actual winner given: CPMW
Fix any voting rule $r$

Input: election

$M \subset V$ given. Question: is $M$ a coalition of possible manipulators?

Actual winner given: CPMW

Actual winner not given: CPM
Problem Definition

Problem Formulation

Fix any voting rule $r$

**Input:** election

- $M \subset V$ given. Question: is $M$ a coalition of possible manipulators?
- Find: a coalition of possible manipulators $M$ with $|M| = k$.

**Actual winner given:** CPMW

**Actual winner not given:** CPM
Problem Formulation

Fix any voting rule $r$

- Input: election

$M \subseteq V$ given. Question: is $M$ a coalition of possible manipulators?
- Actual winner given: CPMW
- Actual winner not given: CPM

Find: a coalition of possible manipulators $M$ with $|M| = k$.
- Actual winner given: CPMSW
Fix any voting rule \( r \)

**Input:** election

\( M \subset V \) given. Question: is \( M \) a coalition of possible manipulators?

Find: a coalition of possible manipulators \( M \) with \(|M| = k\).

- Actual winner given: \( \text{CPMW} \)
- Actual winner not given: \( \text{CPM} \)
- Actual winner given: \( \text{CPMSW} \)
- Actual winner not given: \( \text{CPMS} \)
### Results

<table>
<thead>
<tr>
<th>Voting Rule</th>
<th>CPM, $k = 1$</th>
<th>CPM</th>
<th>CPMW, $k = 1$</th>
<th>CPMW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scoring Rules</td>
<td>P</td>
<td>?</td>
<td>P</td>
<td>?</td>
</tr>
<tr>
<td>Borda</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>P</td>
</tr>
<tr>
<td>$k$-approval</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>P</td>
</tr>
<tr>
<td>Maximin</td>
<td>P</td>
<td>?</td>
<td>P</td>
<td>NPC</td>
</tr>
<tr>
<td>Bucklin</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>P</td>
</tr>
<tr>
<td>STV</td>
<td>NPC</td>
<td>NPC</td>
<td>NPC</td>
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</tr>
</tbody>
</table>

**Borda:** manipulation is NPC [Davies et al., 2011].

**Borda:** Detecting manipulation is easy.
Future Work

- Verifying the number of false positives that this model catches in a real or synthetic data set

- Does any practical model on voters’ true preference reduce false positives?
Questions?

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- MHRD, India, for providing scholarship 😊
