Which Visits Lead to Purchases?
Dynamic Conversion Behavior at e-Commerce Sites

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ABSTRACT:

This paper develops a model of conversion behavior (i.e., converting store visits into purchases) that predicts each customer’s probability of purchasing based on an observed history of visits and purchases. We offer an individual-level probability model that allows for consumer heterogeneity in a very flexible manner. We allow visits to play very different roles in the purchasing process. For example, some visits are motivated by a planned purchase while others are simply browsing visits. The Conversion Model in this paper has the flexibility to accommodate a number of visit-to-purchase relationships. Finally, consumers’ shopping behavior may evolve over time as a function of past experiences. Thus, the Conversion Model also allows for non-stationarity in behavior. Specifically, our Conversion Model decomposes an individual’s purchasing conversion behavior into a visit effect and a purchasing threshold effect. Each component is allowed to vary across households as well as over time. We then apply this model to the problem of “managing” visitor traffic. By predicting purchasing probabilities for a given visit, the Conversion Model can identify those visits that are likely to result in a purchase. These visits should be re-directed to a server that will provide a better shopping experience while those visitors that are less likely to result in a purchase may be identified as targets for a promotion.
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1. Introduction

As it is virtually costless for Internet shoppers to visit an e-commerce site, many online merchants experience large volumes of visitor traffic. Amazon, for example, had approximately 15 million unique visitors at their site just in the month of April 2000 (MediaMetrix 2000), while Victoria’s Secret occasionally experiences huge surges in traffic -- as many as 1000 visitors per second (Quick 1998). Consequently, an important issue facing e-commerce managers is how best to handle the large numbers of shoppers given limited server capacity.

One solution is to invest in server capacity, since too little capacity results in congested server traffic and provides users with a poor shopping experience. But too much capacity is expensive, and large investments in server capacity are poorly rewarded, since conversion rates of visits to purchases are uniformly low across the industry. Forrester (1999) reported that over 70% of online retailers experienced less than a 2% overall purchase conversion rate. In other words, over 98% of the visits that e-commerce sites must serve do not result in purchase transactions. Should e-commerce sites invest to ensure that the 2% of the visits resulting in purchases will be positive shopping experiences or invest less and hope that 2% of the visits will still result in purchases despite any potential negative shopping experiences?

One way e-commerce sites have chosen to deal with this dilemma is to “manage” visitor traffic. For example, Victoria’s Secret re-directs purchasing visits to faster servers while mere browsing visits are hosted by their less efficient and often congested servers (Quick 1998). But purchasing visits are only identified after the shopper places an item in the shopping cart. The shopping experience leading up to that event is still hosted by the slower server. The ability to identify purchasing visits at the start of the session rather than after the customer has finished their shopping and begun to complete the
transaction, would offer a better overall shopping experience for the more promising visits, and therefore might increase the number of visits that proceed towards a successful purchase transaction.

Because of the low returns to serving the large volume of visitor traffic, a related issue is how best to allocate marketing resources in order to improve conversion rates. Many e-commerce sites hold the philosophy that every visit is a buying opportunity and therefore try to induce purchases with promotions and discounts. However, offering promotions and discounts for all visits is inefficient. Many purchasers are likely to initiate a transaction without the added incentive of a promotion; offering these shoppers a promotion would be a poor use of resources.

The key in both of these situations is the ability to predict purchasing probabilities for a given visit. Those visits that are likely to result in a purchase need to be identified and the visitors possibly redirected to a server that will provide a better shopping experience. Those visits that are less likely to result in a purchase, without any added incentive, may be identified as targets for promotion.

With these (and other) resource allocation decisions in mind, we develop a model of conversion behavior that predicts each customer’s probability of purchasing at a given visit based on that individual’s observed history of visits and purchases. We offer an individual-level probability model that allows for consumer heterogeneity in a very flexible way. We discuss and allow for the fact that visits may play very different roles in the purchasing process. For example, some visits are motivated by a planned purchase while others are simply browsing visits. The Conversion Model developed in this paper has the flexibility to accommodate a variety of visit-to-purchase relationships. Finally, consumers’ shopping behavior may evolve over time as a function of past experiences. Thus, the Conversion Model also allows for non-stationarity in behavior.

The next section further illustrates the research problem and some of the dynamics we wish to model. In §3, we review some of the literature covering search and browsing behavior to better understand the different roles that visits can play with respect to purchasing. Then, we develop the Conversion Model in §4 and describe the estimation and data in §5 and §6. After presenting the results
and discussing some of the implications of those results in §7, we demonstrate the value of the model in identifying those visits that are likely to result in a purchases.

2. Illustration of Conversion Behavior Dynamics

To illustrate the research problem, consider individual A characterized by the following pattern of visits and purchases, where $t_{ij}$ denotes the time of individual $i$'s $j^{th}$ visit and P indicates the visits during which a purchase took place. For example, individual A visited a particular store five times prior to $t_{A6}$ and purchased twice, once at $t_{A1}$ and again at $t_{A3}$.

A: $t_{A1}$ $t_{A2}$ $t_{A3}$ $t_{A4}$ $t_{A5}$ $t_{A6}$$\begin{array}{c}
\text{P} \\
\text{P} \\
? \\
\end{array}$

Given that the consumer visits at $t_{A6}$, what is her probability of purchasing during that visit? Now compare this person to individual B, who is characterized by the following history:

B: $t_{B1}$ $t_{B2}$ $t_{B3}$ $t_{B4}$$\begin{array}{c}
\text{P} \\
\text{P} \\
? \\
\end{array}$

Notice that the purchasing history is identical to that of consumer A while visiting behavior is different. At $t_{B4}$, is this individual more or less likely to purchase than person A at $t_{A6}$? The key to this question is in understanding the role of non-purchase visits (e.g., $t_{A4}$ and $t_{A5}$) on purchasing behavior – do these visits increase or decrease the likelihood of a purchase at the next visit?

In addition to the effect of past visits on purchasing, as illustrated by the above examples, past purchasing behavior may also affect future conversion behavior. Consider individual C, who has had a visiting history identical to A.

C: $t_{C1}$ $t_{C2}$ $t_{C3}$ $t_{C4}$ $t_{C5}$ $t_{C6}$$\begin{array}{c}
\text{P} \\
\text{P} \\
? \\
\end{array}$
The only difference between C and A is the timing of past purchases. How will this affect C’s purchasing probability at $t_{C6}$? Since a purchase just occurred at $t_{C5}$, person C may be less likely to purchase at $t_{C6}$. However, individual C could be more likely to purchase if the recent purchase was a positive experience and therefore influential in reducing any reluctance to purchasing yet again.

Furthermore, what if more purchases had been made? How would an added purchase at $t_{C3}$ change this individual’s probability of purchase at $t_{C6}$?

The Conversion Model developed in this research will address three important issues that the above scenarios help illustrate. First, what is the effect of the number and timing of past visits on future purchasing behavior? Second, what is the effect of the number and timing of past purchases on future behavior? And third, do these effects change over time as consumers gain more experience at a particular website?

3. The Role of Visits

To better understand conversion behavior and the role of visits on purchasing, it is useful to review some of the existing literature on consumer search. Search behavior has been dichotomized into goal-directed versus exploratory search (Janiszewski 1998). Goal-directed search refers to visits for which the consumer has a specific or planned purchase in mind. The objective of search in this case is to allow the consumer to gather relevant information for either an immediate or a future purchase (Brucks 1985, Wilkie and Dickson 1985).

Exploratory search, on the other hand, refers to visits for which the consumer is not actively planning a purchase. Instead, search tends to be undirected and stimulus-driven rather than goal-driven (Janiszewski 1998). This type of search is also sometimes referred to as browsing or ongoing search. Since ongoing search is not motivated by any specific decision-making need, the consumer derives utility not necessarily from the outcome of search but rather from the visiting/shopping experience itself (Bloch, Sherrell, and Ridgway 1986).
Both types of search behavior can potentially result in a purchase. In some cases, a visit can have an immediate effect and would result in a purchase at the time of the visit. In other cases, the visit may not result in an immediate purchase but instead may have a longer term effect on future purchasing behavior. Table 1 provides a structure to combine these two dimension (directed vs. exploratory search, short-term vs. long-term effects) to enable us to view the different roles that store visits have on purchasing behavior. From this framework, we can classify the role of visits into one of four categories: (1) directed purchase visits, (2) search/deliberation visits, (3) hedonic browsing visits, and/or (4) knowledge-building visits. Each one of these roles suggests a different relationship between visits and purchases and hence has implications on our ability to understand and capture changes in conversion probabilities over time and across people.

[Table 1. Different Roles of Store Visits]

**Directed-Purchase Visits.** Many visits are driven primarily by directed search for the purpose of an immediate purchase. As a result, the effects of these store visits are realized immediately at the time of the visit. When this is the predominant behavior in the market, visits are strongly related to purchases and conversion rates are very high, as in the case of consumer packaged goods and groceries. In this type of visit-to-purchase relationship, visits generally result in immediate purchase, and therefore, visits can be said to have a high *baseline effect* on stimulating purchases.

**Search/Deliberation Visits.** In other situations, visits are not always accompanied by a purchase even though a purchase is planned. *Search/deliberation visits* are driven by directed search for a planned purchase but not necessarily an immediate one. Instead, the consumer may be considering a future purchase, and the objective of the visit is to acquire and accumulate product related information to help the consumer make a more optimal choice (Punj and Staelin 1983, Putsis and Srinivasan 1994). In these cases, the effect of store visits accumulate towards a purchasing threshold that, when reached, will result in actual purchase.
**Hedonic Browsing Visits.** Both directed-purchase visits and search/deliberation visits are motivated by directed search behavior. However, many store visits are not made with a specific purchase in mind. Instead, consumer shopping behavior is exploratory and any resulting purchase tends to be unplanned prior to the visit. *Hedonic browsing* visits refer to store visits made by a consumer for the hedonic utility of the experience. Many consumers visit stores and shop for pleasure or recreation (Babin, Darden, and Griffin 1994, Hirschman 1984, Sherry, McGrath, and Levy 1993). As a result, these visits tend to be more stimulus-driven and occasionally result in impulse buying, depending on the nature of the stimuli encountered (Janiszewski 1998, Jarboe and McDaniel 1987). In these cases, the impact that visits have on purchasing can be described as being stochastically determined and immediate -- there is no accumulation effects of visits on future purchasing. Therefore, the effect of *hedonic browsing* visits on purchasing will vary across visits but will generally not carry over to affect future purchasing.

**Knowledge-Building Visits.** Finally, there are situations in which search is exploratory in nature but the utility derived from the experience is utilitarian rather than hedonic. For example, ongoing search is frequently motivated by a desire to accumulate a bank of product information that may potentially be useful in the future (Bloch, Sherrell, and Ridgway 1986). These visits increase the consumer’s product and/or marketplace expertise. The consumer is not necessarily considering any specific purchase, but the increased knowledge gained from the store visit may affect long-term shopping behavior; specifically, it reduces the amount of explicit search needed to make future purchasing decisions (Alba and Hutchinson 1987, Bettman 1979).

In the next section of the paper, we lay out the conceptual and mathematical details that come together to form our Conversion Model. As we do so, we will refer back to this visits taxonomy repeatedly in order to illustrate the flexibility of our proposed model, as well as its ability to address the types of managerial decision scenarios described at the outset of the paper.
4. Model Development

Conversion Model Framework. There are four key components of the Conversion Model that provide the capability to accommodate a variety of visit-to-purchase relationships and allow for non-stationary behavior over time:

1. **Baseline probability of purchasing.** For each individual, there is a baseline probability of purchase at each visit. This baseline reflects the extent to which visits are purchase directed and will frequently result in purchasing.

2. **Positive visit effect on purchasing.** Purchasing probability is positively related to the accumulated impact of the store visits made since the shopper’s last purchase. As a shopper makes more visits, she may be more likely to purchase in subsequent visits. Each visit has its own stochastic impact (assumed to be non-negative), and as the effects of these visits accumulate, the probability of purchase increases over time.\(^1\) If and when a purchase occurs, this “bank” of visit effects is reset to zero, and visit effects begin to build up again upon the next visit opportunity.

3. **Negative purchasing threshold effect on purchasing.** Purchasing propensity is negatively related to the individual’s purchasing threshold. As a shopper visits a store, the associated visit effects are measured against this purchasing threshold. When the accumulated effect of visits becomes high relative to the purchasing threshold, purchase occurs. All else equal, the higher an individual’s purchasing threshold, the less likely that customer is to buy.\(^2\)

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\(^1\)This is similar in spirit (but different methodologically) to a model developed by Lenk and Rao (1995) for coupon redemption where the consumer visits the store a number of times before a coupon is redeemed. They conceptualize their model as store visits having effects on the consumer that are stochastically determined and accumulate until coupon redemption occurs. In other words, their model would suggest that as consumers continue to visit store sites, the effect of these visits accumulate and purchasing probabilities change over time. Although their conceptual framework can easily be applied to the relationship between store browsing and purchasing, it does not allow for a single consumer to undergo multiple transitions (i.e., more than one coupon redemption or, in our case, more than one purchase event).

\(^2\) Putsis and Srinivasan (1994) also conceptualize a framework in which buying probabilities are a result of visit effects and purchasing threshold levels, but they focus their attention primarily on a descriptive analysis of the factors that affect pre-purchase deliberation time.
4. Evolving effects over time. The magnitudes of both the visit effects and the purchasing threshold may evolve over time, as the consumer gains experience with the shopping experience. For example, subsequent visits to a website may have smaller effects on purchasing as the shopper gets used to the environmental stimuli and becomes less persuaded by content that she has seen repeatedly in the past. The Conversion Model developed here will allow for and characterize the trends that may exist over time.

**Beta-Bernoulli Model.** We begin the formal specification of our Conversion Model by first introducing the simple beta-Bernoulli process, which is a natural benchmark model for us to build upon. The beta-Bernoulli, when applied to visiting and purchasing behavior, assumes that at any given visit, an individual $i$ has a probability, $p_i$, of purchasing. Additionally, it assumes that these individual purchasing probabilities are beta distributed across the population to account for consumer heterogeneity.

$$f(x_i | p_i) = p_i \quad \text{and} \quad g(p_i) = \frac{1}{B(a,b)} p_i^{a-1} (1-p_i)^{b-1} \quad (1)$$

where $B(a, b)$ is the beta function, $B(a,b) = \frac{\Gamma(a) \Gamma(b)}{\Gamma(a+b)}$. Therefore, the unconditional probability of an individual buying at a given visit is then:

$$f(x) = \int_0^1 p_i \cdot g(p_i) dp_i = E[p_i] = \frac{a}{a+b} \quad (2)$$

If a consumer makes a sequence of visits, the probability of buying at each visit is typically assumed to be the same across visits, i.e., purchasing probability is a latent consumer trait that may vary across consumers but is stationary over time. If this assumption of stationarity were to hold (an assumption that the Conversion Model will later relax) purchasing behavior would not evolve over time, and past visits and purchases would have no impact on latent purchasing probabilities. But even in this stationary environment, our estimate of an individual’s purchasing probability would be updated from visit to visit, using Bayes theorem, as we observe the outcome of each visit as an updated beta distribution with expectation:
where $x_{ij}$ is the number of prior visits resulting in a purchase and $n_{ij}$ is the total number of prior visits.

An important characteristic of a beta random variable is that it can be expressed as a ratio of gamma random variables. Specifically, if $X$ and $Y$ are both gamma distributed random variables such that $X \sim \text{gamma}(a, \gamma)$ and $Y \sim \text{gamma}(b, \gamma)$, then the fraction $X / (X+Y)$ is by definition beta distributed with parameters $a$ and $b$ (Johnson, Kotz, and Balakrishnan 1995, p.350). This is a fundamental relationship which will help us develop our complete Conversion Model.

**Conversion Model.** Starting with the basic beta-Bernoulli structure, we bring in the various elements of the aforementioned conceptual framework in the following manner. Let $p_{ij}$ be the probability of individual $i$ purchasing at visit $j$ (where $j=0$ is the initial visit observed in a particular dataset). This probability is a function of both visit impacts as well as a purchasing threshold. That is, each time a consumer visits, there is a subsequent impact that may accumulate toward a purchasing threshold. It follows that the individual’s purchasing probability is determined by the net effect of these visit impacts relative to this threshold.

\[
p_{ij} = \frac{\text{Net Effect of Visits Since Last Purchase} (V_{ij})}{\text{Net Effect of Visits Since Last Purchase} (V_{ij}) + \text{Purchasing Threshold} (\tau_{ij})}
\]  

(4)

We assume that the net visit effect, $V_{ij}$, consists of two components; a baseline propensity to buy ($v_{i0}$) that applies at every visit and the incremental effects ($m_{ij}$) from the visits that have occurred since the last purchase. Thus, $V_{ij}$ is the sum of the baseline effect and all qualifying visit effects. In an individual’s first purchasing cycle, for example, $V_{ij}$ is equal to $v_{i0} + m_{i0} + m_{i1} + \ldots + m_{ij}$ for household $i$ who has made $j+1$ visits. A large baseline effect relative to the magnitude of visit impacts would indicate that a given consumer tends to be a directed purchase visitor. On the other hand, if the impact of visits is large
relative to the baseline effect, the consumer may be highly influenced by the in-store environment, as is the case with *search/deliberation* visits and *hedonic browsing* visits.

If individual $i$ has a purchasing threshold, $\tau_{ij}$, her purchasing probability can then be written as:

$$p_{ij} = \frac{v_{i0} + m_{i0} + m_{i1} + \ldots + m_{ij}}{v_{i0} + m_{i0} + m_{i1} + \ldots + m_{ij} + \tau_{ij}}$$

(5)

To accommodate heterogeneity, let us assume that the baseline purchasing propensity $v_{i0}$ is gamma distributed across the consumer population with shape parameter $r_v$ and scale parameter $\gamma$. Let us also assume that the visit impacts, as well as purchasing thresholds, are heterogeneous and gamma distributed such that $m_{ij} \sim \text{Gamma}(\mu, \gamma)$ and $t_{ij} \sim \text{Gamma}(r, \gamma)$.

The resulting purchasing probability is then a ratio of two gamma random variables which, by definition, is a beta-distributed random variable.

**Evolving Visiting Effects.** Thus far, we have assumed that the gamma distribution governing the impact of visits remains stationary over time with parameters $\mu$ and $\gamma$. In other words, while the impact of successive visits may vary over time, we have not allowed for any trends in this stochastic process. However, it is likely that, as consumers familiarize themselves with the store site, visit effects ($m_{ij}$) might change systematically. We therefore extend the model to allow for the possibility that the influence of store visits will increase, decrease, or stay the same depending on how familiar the individual is with the site. We approximate the consumer’s experience with the site with the number of times she has previously visited the site and implement the evolutionary trend through the shape parameter governing the incremental visit effects, $m_{ij}$ (Schmittlein and Morrison 1998). Therefore, we assume:

$$m_{ij} \sim \text{Gamma}(\mu_j, \gamma) \quad \text{where} \quad \mu_j = \mu_0 k^j$$

(6)

and thus the net effect of visits for the first purchase cycle then becomes:

$$V_y \sim \text{Gamma}(r_v, \gamma) + \text{Gamma}(\mu_0 k^0, \gamma) + \text{Gamma}(\mu_0 k^1, \gamma) + \ldots + \text{Gamma}(\mu_0 k^j, \gamma)$$

(7)

\[5\] Since the sum of the baseline effect and the visit impacts are compared against the threshold to determine purchasing propensity, it is reasonable to assume that they are measured on the same scale, and thus the distributions governing these three effects share the same scale parameter $\gamma$. 

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The parameter \( k \) ranges from 0 to infinity and characterizes how visit impacts evolve as consumer familiarity increases. If \( k \) equals 1.0, there is no evolutionary effect; the stochastic process governing \( m_{ij} \) is a simple stationary one. If \( k \) is less than 1.0, visits tend to become less influential over time, while if \( k \) is greater than 1.0, visits tend to become more influential as consumers evolve. But despite the upward or downward trend on the shape parameter, each successive draw of \( m_{ij} \) is still a random variable, thereby allowing any particular visit to have an unusually high or low impact. This allows for the possibility of impulse purchases, albeit with different probabilities, at any given visit.

Another issue to be considered is the timing of past visits and their impact on purchasing in the current visit. A visit that occurred six months ago may not have the same effect in stimulating a purchase today as a visit that occurred just last week. Therefore, we weight the impact of each visit depending on how much time has past since that visit occurred. Consistent with our implementation of evolving visit impacts, we implement this weighting process through the shape parameter:

\[
\mu_u = \mu_0 k^u \delta_v^{t_v - t_u} \tag{8}
\]

where \( \mu_u \) is the shape parameter governing the impact that past visit \( u \) has on current value of \( V_{ij} \), \( t_{ij} (t_u) \) is the time of household \( i \)'s \( j \)th (\( u \)th) visit and \( \delta_v \), ranging from 0 to 1, is the discount rate or importance weight of past visits. The closer \( \delta_v \) is to 1, the greater the effect that past visits have on future behavior, suggesting that visit effects accumulate (as in the case of search/deliberation visits). As \( \delta_v \) approaches zero, consumers tend not to consider past visits. In that case, the effects of visits are seen only in the short-term, as in the case of directed-purchase visits or hedonic browsing visits. In this case, \( V_{ij} \) would be driven only by the gamma-distributed magnitude of the current \((j+1)^{\text{st}} \) visit alone.

For a shopper’s first purchasing cycle, the combination of these effects can be expressed as:

\[
V_{ij} \sim \text{Gamma}(r, \gamma) + \text{Gamma}(\mu_0 k^0 \delta_v^{t_v - t_u}, \gamma) + \text{Gamma}(\mu_0 k^1 \delta_v^{t_v - t_u}, \gamma) + \ldots + \text{Gamma}(\mu_0 k^f \delta_v^{t_v - t_u}, \gamma) \tag{9}
\]

Across multiple purchases, this generalizes to:
where \( lp \) indicates the visit during which the last purchase occurred. If household \( i \) has not yet been observed to make a purchase, then all past visits would contribute to \( V_{ij} \), or \( lp = -1 \). In summary, we assume that the net visit impact is driven by a sum of gamma-distributed random variables yielding a net visit impact that is itself gamma distributed.

The model accommodates three different roles that visits have on purchases: (1) directed-purchase visits, (2) search/deliberation visits, and (3) hedonic browsing visits. Shopping behavior dominated by directed-purchase visits will be associated with a high baseline parameter, \( r_\tau \), relative to the visit impact parameter \( \mu_0 \) as these visits nearly always result in purchase. On the other hand, if consumers are making search/deliberation visits, the visit impact parameter, \( \mu_0 \), will be higher and the discount factor, \( \delta_v \), will be close to one. This allows for lingering effects of visits that contribute to pre-purchase deliberation. However, if consumers are making hedonic browsing visits to stores, the visit parameter, \( \mu_0 \), will also be high while the discount factor, \( \delta_v \), will be close to zero. This allows for stochastic effects of visits that do not carry over but can vary from visit to visit, thereby inducing impulse buying on some occasions.

**Evolving Purchasing Thresholds.** Having completed our discussion of how we model visit effects, i.e., the numerator of equation (4), we now turn our attention to the remaining term in the denominator regarding purchasing thresholds. Initially, the purchasing threshold for household \( i \) is \( \tau_{i0} \), which we specify as a gamma distributed random variable with shape parameter \( r_\tau \) and scale parameter \( \gamma \) to account for consumer heterogeneity. However, over time, a household’s purchasing threshold may evolve depending on the consumer’s past behavior, specifically, their past purchasing experiences. For example, Beatty and Ferrell (1998) found that consumers are more likely to purchase in the future if they have already purchased from that vendor in the past. Conceptually, initial purchasing thresholds may change over time as a result of purchasing experiences. We operationalize this trend in purchasing
thresholds as an exponential function dependent on the number and timing of past purchases and an evolutionary parameter, $\psi$. In much the same way that visit impacts can evolve over time, we implement the evolution of purchasing thresholds through the shape parameter:

$$
\tau_y \sim \text{Gamma} \left( r_\tau \exp \left\{ \psi \cdot \text{purchasing experience} \right\}, \gamma \right)
$$

(11)

where $r_\tau$ is the value of the initial purchasing threshold. The functional form of the shape parameter allows thresholds to either increase, decrease, or remain constant depending on the direction of the evolutionary parameter, $\psi$. If $\psi$ equals zero, the purchasing threshold distribution remains stationary with a shape parameter of $\tau_y$ regardless of past purchasing experiences. If, however, $\psi$ is less than zero, thresholds decline as consumers gain purchasing experience with the retailer, and shoppers become more likely to buy at any given visit. Additionally, the exponential functional form allows for steep changes in the threshold initially but levels off in the longer term. For example, the first few successful purchases may drop the threshold dramatically, but subsequent purchasing might have smaller effects.

We characterize an individual’s purchasing experience in terms of the number and timing of past purchases.

$$
\tau_y \sim \text{Gamma} \left( r_\tau \exp \left\{ \psi \cdot \sum_{q=1}^{t-1} I_{i, \delta_{\tau}^{t-q}} \right\}, \gamma \right)
$$

(12)

where $I_{i, q}$ is a purchase indicator variable for household $i$ at visit $q$ (and $I_{i, 1} = 0$) and $\delta_\tau$ is the discounting factor for past purchases. The discount factor, $\delta_\tau$, ranges from 0 to 1 and weights the role of past purchases on purchasing thresholds. This component of the model can also allow for more regular purchasing behavior. For example, if buying tends to occur at regular time intervals, purchasing thresholds should increase immediately following a purchase and gradually decline afterwards as time passes. This will occur if the evolutionary parameter of the purchasing threshold, $\psi$, is greater than one and the discounting factor, $\delta_\tau$, is small. That is, the more time that has passed since the last purchase, the smaller is the discounted value of past purchases. This would result in a lower purchasing threshold.
even if $\psi > 0$. The combination of the discounting factor and a positive evolutionary parameter allows the purchasing threshold to increase immediately following a purchase and decrease after.

5. Estimation

Two key components determine an individual’s purchasing probability at a given visit, (1) the net effect of past visits and (2) purchasing thresholds. Both of these components are assumed to be gamma distributed random variables to account for heterogeneity. Consequently, the probability of purchase is calculated as $V_y/(V_y+\tau_y)$, where $V_y \sim \text{Gamma}(Q,\gamma)$ and $\tau_y \sim \text{Gamma}(R,\gamma)$, and therefore is Beta distributed with parameters $Q$ and $R$.

To help illustrate the model estimation, consider customer A described in §2. With no prior information, an initial estimate of the buying probability at visit $t_{A1}$ would simply be:

$$p_{A1} \sim \text{Beta}(Q_{A1}, R_{A1}) \quad \text{and} \quad E[p_{A1}] = \frac{Q_{A1}}{Q_{A1} + R_{A1}}$$

where

$$Q_{A1} = r_v + \nu$$

$$R_{A1} = r_s$$

But after observing a purchase at that time, we can update the buying probability much like we do in (3) for the beta-Bernoulli model. But in addition to Bayesian updating, which occurs even in a stationary environment, the shopper’s fundamental behavior is also evolving from visit to visit. This is captured by changes in the Beta parameters, $Q_y$ and $R_y$, according to (10) and (12). The probability that consumer A buys during her second visit then becomes:

$$p_{A2} \sim \text{Beta}(Q_{A2}+1, R_{A2}+1) \quad \text{and} \quad E[p_{A2}] = \frac{Q_{A2}+1}{Q_{A2} + R_{A2} + 1}$$

where

$$Q_{A2} = r_v + \nu \delta k$$

$$R_{A2} = r_s \exp \left( \psi \delta \nu \gamma \right)$$

If the second visit ends without a purchase, then the probability of buying in the third visit, after having observed the two previous visit outcomes, is updated as follows:
Since a purchase occurred at the third visit,

\[
p_{A3} \sim \text{Beta}(Q_{A3}+1, R_{A3}+2) \quad \text{and} \quad E[p_{A3}] = \frac{Q_{A3}+1}{Q_{A3}+R_{A3}+2}
\]

where \( Q_{A3} = r_v + \mu_0 k^2 + \mu_0 k \delta_v t_{A3} \)
\( R_{A3} = r_v \exp \left( \psi \delta_v t_{A3} \right) \) \hspace{1cm} (15)

From this inductive illustration, we jump right to the complete likelihood function, recognizing that, for notational convenience (as shown in (2)), we can use the expressions for \( f(x_i) \) and \( E[p_i] \) interchangeably.

\[
p_{A4} \sim \text{Beta}(Q_{A4}+1, R_{A4}+2) \quad \text{and} \quad E[p_{A4}] = \frac{Q_{A4}+2}{Q_{A4}+R_{A4}+3}
\]

where \( Q_{A4} = r_v + \mu_0 k^3 \)
\( R_{A4} = r_v \exp \left( \psi \left( \delta_v t_{A3} + \delta_v t_{A4} \right) \right) \) \hspace{1cm} (16)

One final component of the model allows for a segment of "hard-core never buyers." In many situations, there is a set of shoppers who visit a store to look around but have absolutely no intention of ever buying anything there. We will assume that these shoppers comprise a fraction, \((1-\pi)\), of the population. This parameter will affect our expectation of \( p_{ij} \) such that if a consumer's history contains no purchases \((x_{ij} = 0)\),

\[
E[p_{ij} | x_{ij} = 0] = (1-\pi) + \pi \frac{Q_{ij}}{Q_{ij} + R_{ij} + n_{ij}}
\]

If, however, consumer \( i \) had purchased in the past and therefore proven herself not to be a never buyer, then the expected probability of buying for all visits following that purchase would be:
\[ \mathbb{E}[P_y | x_y > 0] = \frac{Q_y + x_y}{Q_y + R_y + n_y} \]

6. Data

We use clickstream panel data collected by Media Metrix, Inc. Media Metrix maintains a panel of approximately 10,000 households whose Internet behavior is recorded over time. The data contains information regarding when each household visits a given site. Additionally, the data include the precise day and time when a specific URL was viewed. To consolidate the data, we aggregate visits to the daily level. Any session in which the individual views a URL with the on-line store’s domain name is considered a visit to that store. If a given household visited a store multiple times in a single calendar day, that is coded as one visit on the day when the session began. For our purposes, we examine the panel’s shopping behavior at a leading online bookstore, Amazon. From March 1, 1998 through October 31, 1998, there were 4,379 panelists who made at least one visit making a total of 11,301 visits.

Purchase is defined as any visit during which a purchase occurred. For many online stores, the consumers are linked to a specific web page that acts as a receipt or purchase confirmation after an order has been submitted, including “one-click” purchases. Those visits where the individual saw the “confirm-order” page of the store’s website were identified as purchase visits. The number of units purchased and the total amount spent were not considered in this analysis. Of the 4,379 panelists who visited the store in our data period, only 851 households bought during that time, but they made a total of 1,573 purchases. This results in an overall conversion rate of 13.92% (1,573/11,301) -- a very high conversion rate for online retailers. However, this overall conversion rate says nothing about how buying propensities differ across shoppers or how it changes over time for a given shopper.

Table 2 summarizes the visiting and purchasing behavior at the online bookstore and how it appears to be changing over time. All measures seem to indicate that site performance is improving from the first four months to the next. The conversion rate is increasing, the number of visitors as well as buyers is increasing, and so on. But these aggregate measures are misleading as the inflow of new
shoppers and the dropout of existing shoppers may mask the true underlying dynamics that are occurring at the individual level (Moe and Fader 2000). If we examine the same statistics while accounting for the entering and exiting of shoppers, a very different pattern emerges. Table 3 shows the conversion rate statistics for only those shoppers who appeared to be active at the store throughout the entire data period, namely those who made a visit to the store in both the first two months and the last two months of the data period. This illustrative sub-sample avoids any problems due to censoring and thus provides a better view of individual-level dynamics. The behavior of these households during the four intermediate months are then compared.

[Table 2. Summary of Visiting and Purchasing]

[Table 3. Conversion Rate Summary for Active Shoppers Only]

In general, conversion rates for this subset of the population are much higher than those seen in the population as a whole as more active shoppers tend also to be more likely buyers (Moe and Fader 2000). But, contradicting the increasing conversion rates for the entire sample as seen in Table 2, this group’s conversion rates are actually decreasing over time. Therefore, without modeling behavior at the individual level, e-commerce managers may be drawing incorrect conclusions. The Conversion Model is able to capture these individual-level patterns, and therefore provides a better indication of differences across households as well as the dynamics over time.

7. Results

As a start, the full eight parameter Conversion Model was estimated on the entire eight months of bookstore data (see Table 4, row 1). From the model results, there are four main dynamics we are looking to identify. First is the influence of visits. Do visits have some effect on conversion probabilities or is most purchasing driven by a strong baseline effect in the case of directed purchase
visiting behavior? The model seems to indicate that, for the most part, consumers are not directed buyers since baseline purchasing is very low as indicated by the negligible value of $r_v$.

| Table 4. Parameter Estimates for CR and nested models |

Second is the adaptation effect. That is, does the incremental effect of each visit systematically evolve as the shopper gains experience? In this case, $k$ is less than one suggesting that subsequent visits have a diminishing impact on purchasing behavior as the shopper makes more visits to the site.

Third, how do past visits affect current conversion probabilities? There are two components to this issue. The first is to examine whether or not visit effects accumulate from visit-to-visit, as suggested by Lenk and Rao (1995) and Putsis and Srinivasan (1994). The key parameter here is the discount rate ($\delta_r$). From the full model, it appears that older visits are discounted only slightly ($\delta_r=0.999$) if at all. The significance of this parameter (as well as others) will be tested by estimating a number of nested models. However, it is clear from the full model that visit effects do accumulate, and they may not necessarily be discounted.

Changes in purchasing thresholds, or the effect of past purchases, is the final dynamic that we examine. From the full model estimated, it seems that purchasing thresholds increase as a function of discounted purchasing experiences ($\psi=0.381$), and thus a consumer is less likely to re-purchase soon after a transaction occurs. This is very discouraging to many online retailers who believe that online purchasing experiences will be so positive that future purchasing will be more likely. But it is important to note that purchasing thresholds are a function of discounted past purchases. A purchasing discount rate ($\delta_p$) less than one could diminish the effect of this increased threshold, thereby making a future purchase fairly likely. But in this case, past purchases are discounted so little ($\delta_v=0.984$) that purchasing thresholds are unlikely to return to the original lower levels that encourage purchasing within any reasonable time frame.
Overall, conversion probabilities are driven largely by the accumulation of visit effects suggesting that the shopping process is one of search/deliberation. There are also interesting dynamics occurring among the consumers over time. Visits have a smaller effect on purchasing probabilities as consumers adapt to the shopping environment. Also, purchasing thresholds are increasing as a function of past purchasing experiences. Both of these dynamics suggest that conversion probabilities are decreasing over time, as a function of past visit experiences and as a result of increased purchasing experiences -- contradicting the trends suggested by the aggregate conversion rate measures (but supporting the type of data pattern shown in Table 3). This bodes very poorly for the future of this website if such dynamics continue.

Table 4 shows the parameter estimates and log-likelihoods of several nested models, including the static, two-parameter, beta-Bernoulli model. Additional likelihood ratio tests were conducted to identify the model that fit significantly better than the others given the number of parameters it used. Though several models were comparable in terms of BIC, the best model (model #6) according to likelihood ratio tests is that with only one fixed parameter, \( \delta_v = 1 \), suggesting that there is no discounting of past visit effects, i.e., older visits are just as impactful as more recent visits. The remaining parameter estimates are very similar to those of the full eight-parameter model leading to the same conclusions and implications as discussed above.

7. Identifying High Purchasing Probability Visits

Having conceptualized, developed, and estimated a model that accommodates the different behavioral processes discussed in §2, we now return to the resource allocation decisions that motivated this modeling exercise. Our goal in this section is to evaluate the model’s ability to accurately identify likely purchase visits based only on the visitor’s behavioral history to date. This will help the e-
commerce manager target the appropriate customers who may benefit from having access to faster
servers as well as those who may be good (or bad) targets for short-term promotional efforts.

To test the value of using the Conversion Model to identify high purchasing probability visits, we
use the first four months of data to calibrate the model. We then estimate each shopper’s expected
conversion probability for their first visit in the forecasting period. Based on these projected conversion
probabilities, we rank and identify the top 10% of customers as having the highest probabilities of
purchase at that point in time. We use two separate benchmark models with which to compare the
Conversion Model projections.

The first benchmark is a simple projection of historical (static) conversion rates -- the current
standard among e-commerce managers. An individual’s projected future conversion probability is
simply the total number of past purchases in the calibration period divided by the total number of visits in
the same period. The second benchmark model is the beta-Bernoulli which is more sophisticated than
the historical conversion rate as it allows for consumer heterogeneity. Each household’s projected future
conversion probability is a shrinkage estimate (3) based on both the parameters of the beta distribution
and each household’s own distinct visiting and purchasing history. With each method, customers are
ranked and the top 10% are identified as having high probabilities of purchase in the next visit.

The top 10% according to the Conversion Model are also identified. Unlike the projections
using the two benchmark models, this requires the use of a simulation. Since the Conversion Model
addresses the dynamics that occur from visit to visit, it is not as simple as calculating just one number
based on historical data and projecting that number forward for all future visits made by that individual.
Using the resulting parameters estimates of the model, we simulate future purchasing probabilities for the
first visit in the forecasting period based on the number and timing of visits and purchases observed in
the calibration period. This simulation is repeated 100 times. The expected conversion probability for
an individual’s next visit is the average number of purchases for that visit across all 100 iterations of the
simulation. Customers are then ranked according to these conversion probabilities, and the top 10% are identified.

To assess the value of using each model in identifying this select group of customers, the actual future conversion rates of the top 10%, as designated by each method, are compared (Table 5). The top 10%, according to projections based on historical conversion rates, result in only a 29.3 purchases per 100 households. Though this conversion rate is better than the 14.0% for the population as a whole, it is inferior to the conversion rates that result from using either the beta-binomial (33.0%) or the Conversion Model (37.3%). If an e-commerce manager targets the high purchasing probability customers as identified by the Conversion Model, the return in terms of conversion rates is 12.8% higher than targeting those identified by the static beta-binomial model.

Table 5. The Top 10%'s Actual Conversion Rates

9. Conclusions

The Internet has provided e-commerce managers with an over-abundance of data and metrics. The objective of this paper was to investigate one of these metrics – conversion rates. Specifically, the model presented in this paper allows for a more careful and useful examination of conversion behavior than a simple aggregation of the number of visits and purchases can provide. We have illustrated that aggregate summary measures often provide misleading results and conclusions. The Conversion Model avoids this mistake by directly addressing heterogeneity across consumers as well as dynamics over time. Because consumers have different reasons for visiting a web store, it is important to understand and account for these various patterns of visiting and purchasing. These patterns are often overlooked but are addressed carefully in our Conversion Model.

Understanding how conversion probabilities vary across consumers and change from visit to visit is valuable information that can allow e-commerce managers to better treat each visitor based on his/her past behavior. The patterns of visits and purchases may reveal how best to serve a given visit. E-
commerce managers should ensure that the high purchasing probability visits are positive shopping experiences. Those visits less likely to result in a purchase may be prime targets for a promotion.

The research problem of examining conversion probabilities is very complex, and several issues are still unexplored. For example, we have ignored the activity within each visit. The sequence of pageviews (e.g., duration, type of pages examined, etc.) could have a great influence on changing conversion probabilities and the likelihood that a consumer will buy in any given visit. Once the process of evolving conversion behavior within a single visit is better understood, we can then begin to investigate the effectiveness and efficiency of promotional tools offered at different moments of the consumer’s store visit.

Methodologically, the Conversion Model allows for heterogeneity and various different effects of store visits. However, it is unable to characterize each individual by their dominant shopping behavior, i.e., searcher, browser, directed buyer, etc. Therefore, one area for potential future research is to extend this model to utilize hierarchical Bayesian methods. This would allow marketers to differentiate among different types of shoppers and thus design different marketing campaigns to better suit each consumer.

Additionally, this paper has focused exclusively on conversion probabilities without addressing the issue of modeling visiting patterns. Such a model is presented in Moe and Fader (2000), but a promising future project would be to integrate these two approaches to obtain a more complete picture of online visit-purchase behavior. In combining these two models, we can begin to investigate the efficiency of marketing dollars aimed at increasing visits versus encouraging conversion behavior. Such insights would be of tremendous value to practitioners. But before any comprehensive optimization schemes can be achieved, it is important that we first gain a complete, theoretically grounded, understanding of each of the processes that underlie the observable behaviors. We believe that this model is a useful first step in this direction, and we encourage future researchers to build upon it.
REFERENCES


### Table 1. Different Roles of Store Visits

<table>
<thead>
<tr>
<th>Role</th>
<th>Short Term or Immediate Visit Effect</th>
<th>Long Term, Accumulating Visit Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directed Search</td>
<td>Directed Purchase Visit</td>
<td>Search/Deliberation Visit</td>
</tr>
<tr>
<td>Exploratory/Ongoing Search</td>
<td>Hedonic Browsing Visit</td>
<td>Knowledge-Building Visit</td>
</tr>
</tbody>
</table>

### Table 2. Summary of Bookstore Visiting and Purchasing

<table>
<thead>
<tr>
<th></th>
<th>All 8 months</th>
<th>Months 1-4</th>
<th>Months 5-8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of visitors</td>
<td>4,379</td>
<td>2,693</td>
<td>2,717</td>
</tr>
<tr>
<td>Number of buyers</td>
<td>851</td>
<td>468</td>
<td>531</td>
</tr>
<tr>
<td>Number of visits</td>
<td>11,301</td>
<td>5,402</td>
<td>5,899</td>
</tr>
<tr>
<td>Number of purchases</td>
<td>1,573</td>
<td>705</td>
<td>868</td>
</tr>
<tr>
<td>Conversion rate</td>
<td>13.92%</td>
<td>13.05%</td>
<td>14.71%</td>
</tr>
<tr>
<td>Visits/visitor</td>
<td>2.58</td>
<td>2.01</td>
<td>2.17</td>
</tr>
<tr>
<td>Purchases/buyer</td>
<td>1.85</td>
<td>1.51</td>
<td>1.63</td>
</tr>
<tr>
<td>Purchases/visitor</td>
<td>0.36</td>
<td>0.26</td>
<td>0.32</td>
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</tbody>
</table>

### Table 3. Conversion Rate Summary for Active Bookstore Shoppers Only

<table>
<thead>
<tr>
<th></th>
<th>Months 3-4</th>
<th>Months 5-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conversion rate</td>
<td>26.02%</td>
<td>20.76%</td>
</tr>
<tr>
<td>Number of visits</td>
<td>757</td>
<td>472</td>
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<tr>
<td>Number of purchases</td>
<td>197</td>
<td>98</td>
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</table>
**Table 4. Parameter Estimates for Conversion and Nested Models at Bookstore**

<table>
<thead>
<tr>
<th></th>
<th>( r_v )</th>
<th>( \mu )</th>
<th>( k )</th>
<th>( \delta_v )</th>
<th>( r_\tau )</th>
<th>( \psi )</th>
<th>( \delta_\tau )</th>
<th>( \pi )</th>
<th>LL</th>
<th>BIC</th>
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</thead>
<tbody>
<tr>
<td>1. Full model</td>
<td>0.000</td>
<td>0.235</td>
<td>0.934</td>
<td>0.999</td>
<td>1.747</td>
<td>0.381</td>
<td>0.984</td>
<td>0.814</td>
<td>-4261.05</td>
<td>8589.18</td>
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<tr>
<td>2. Visit dynamics</td>
<td>0.134</td>
<td>0.305</td>
<td>0.920</td>
<td>0.999</td>
<td>3.139</td>
<td>0</td>
<td>0</td>
<td>0.785</td>
<td>-4266.03</td>
<td>8582.37</td>
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<tr>
<td>3. Threshold dynamics</td>
<td>0.362</td>
<td>0.362</td>
<td>1</td>
<td>0</td>
<td>4.742</td>
<td>0.000</td>
<td>0.000</td>
<td>0.918</td>
<td>-4307.90</td>
<td>8649.34</td>
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<td>4. No discounting</td>
<td>0.062</td>
<td>0.257</td>
<td>0.912</td>
<td>1</td>
<td>2.314</td>
<td>0.117</td>
<td>1</td>
<td>0.790</td>
<td>-4264.25</td>
<td>8578.81</td>
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<td>5. Full discounting</td>
<td>0.000</td>
<td>0.743</td>
<td>1.034</td>
<td>0</td>
<td>5.156</td>
<td>-0.289</td>
<td>0</td>
<td>0.919</td>
<td>-4298.20</td>
<td>8646.71</td>
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<td>6. No visit discounting*</td>
<td>0.000</td>
<td>0.229</td>
<td>0.931</td>
<td>1</td>
<td>1.697</td>
<td>0.396</td>
<td>0.984</td>
<td>0.807</td>
<td>-4261.18</td>
<td>8581.05*</td>
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<tr>
<td>7. No visit effect evolution</td>
<td>0.099</td>
<td>0.436</td>
<td>1</td>
<td>0.933</td>
<td>4.047</td>
<td>0.000</td>
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<td>0.904</td>
<td>-4279.06</td>
<td>8600.04</td>
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<td>8. Visit dynamics, no discounting</td>
<td>0.155</td>
<td>0.291</td>
<td>0.912</td>
<td>1</td>
<td>3.159</td>
<td>0</td>
<td>0</td>
<td>0.777</td>
<td>-4266.17</td>
<td>8574.26</td>
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<td>9. No purchase discounting</td>
<td>0.043</td>
<td>0.269</td>
<td>0.933</td>
<td>0.999</td>
<td>2.295</td>
<td>0.118</td>
<td>1</td>
<td>0.798</td>
<td>-4264.07</td>
<td>8586.83</td>
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<td>10. Static beta-binomial</td>
<td>0.613</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>4.436</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-4308.03</td>
<td>8632.83</td>
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</tbody>
</table>

NOTE: The numbers in bold indicate the values at which the parameters were fixed.  
* Selected Model

**Table 5. The Top 10%’s Actual Conversion Rates at Bookstore**

<table>
<thead>
<tr>
<th>Top 10% according to...</th>
<th>Actual Conversion Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical Conversion Rates</td>
<td>29.3%</td>
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<tr>
<td>Beta-Binomial Model</td>
<td>33.0%</td>
</tr>
<tr>
<td>Conversion Model</td>
<td>37.3%</td>
</tr>
</tbody>
</table>