Almost Budget Balanced VCG Mechanisms

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Agenda

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  - Groves : Incentive in Teams.
  - Groves Mechanism.
  - Some Possibility and Impossibility Results.

Part II

- Almost Budget Balanced VCG Mechanisms
  - Scenario Under consideration.
  - Efficiency Loss.
  - Optimal Mechanisms.

- Open Problems

Problem

Consider a conglomerate-type organization consisting of a set of semi-autonomous subunits that are coordinated by the organization's head. The head's incentive problem is to choose a set of employee compensation rules that will induce his subunit managers to communicate accurate information and take optimal decisions. (T Groves [1])

Notation

N	$=$	The set of agents $\{0, 1, 2, 3, ..., i, ..n\}$ 0 : The head. $1, 2, \ldots n$: Subunit Managers
Θ_i	$=$	Type profile of an agent i
θ_i	$=$	Type of agent i
Θ	$=$	$\prod_{i=0}^{i=n} \Theta_i$
θ	$=$	$(\theta_0, \ldots, \theta_i, \ldots, \theta_n)$
S_i	$=$	Strategy Space of the agent i
S	$=$	$\prod_{i=0}^{i=n} S_i$
s	$=$	$(s_0, \ldots, s_i, \ldots, s_n)$

u_i	:	$Sx\Theta \rightarrow \mathbb{R}$ The Pay off function to agent i.
T	$=$	$\{N, \Theta, \{S_i : i \in N\}, u_0\}$ The Team Model.
G	$=$	$\{N, \Theta, \{S_i : i \in N\}, \{u_i : i \in N\}\}$ The Induced Game .
$(\hat{s}/s_i)$	$=$	$(\hat{s_0}, \ldots \hat{s_{i-1}}, s_i, \hat{s_{i+1}}, \ldots \hat{s_n})$

Table: Notation Incentive in Teams Problems

The Team's Problem

The team's objective is assumed to be to choose a joint strategy $s^* \in S$, if one exists, that maximizes the expected value of the team payoff function u_0.

$$E[u_0(s^*)] \equiv \max_{s \in S} E[u_0(s)]$$

Assumption A

There exists a $s^* \in S$ such that

i. $E[u_0(s^*)] \geq E[u_0(s)] \;\; \forall\, s \;\in\; S$

ii. $E[u_0(s^*)] > E[u_0(s^*/s_i)] \;\; \forall\, s_i \neq {s_i}^*,\; s_i \in S_i \;\forall i \;\in\; N$

Incentive Problem

- A set $U = \{u_i : i = 1, \ldots, n\}$ of employee payoff functions is called an *incentive structure*, and an incentive structure $U^* = \{u_i^*, i = 1, \ldots, n\}$ is called *optimal* if, for the joint strategy s^* satisfying Assumption,

$$E[u_i^*(s^*)] = \max_{s_i \in S_i} E[u_i^*(s^*/s_i)] \text{ uniquely for all } i = 1, \ldots, n.$$

- The incentive problem of the organization head is then to find an optimal incentive structure U^*, or equivalently, since every incentive structure U defines an $(n+1)$-person game, to choose the optimal game for his organization to play.

Incentive Structure : Two Common Systems

1. *'Paid worker incentive structure.'*
   Define, $U^0 = \{u_i^0, i = 1, \ldots, n\}$ by,

$$u_i^0(s,\theta) = \begin{cases} 1 & : \text{ if } s_i = s_i^* \\ 0 & : \text{ otherwise} \end{cases}$$

   Disadvantage : Assumes complete information setting.

2. Define, $U^I = \{u_i^I, i = 1, \ldots, n\}$ by, $(i = 1, \ldots, n)$

$$u_i^I(s,\theta) = \alpha_i * u_0(s,\theta) + A_i$$

   where, α_i is positive constant and A_i any constant.
   This is called *'profit sharing incentive structure'.*

   Disadvantage : Does not discriminate between optimally and non-optimally performing employees.

Model of conglomerate

Specifications

1. $$u_0(s,\theta) = \sum_{i=1}^{n} v_i[s,\theta_i] + v_0[s,\theta_0]$$

2. *The portion $v_i[s,\theta_i]$ of the organization payoff accrues directly to the i^{th} subunit.*

- $\mathscr{I} = [U = \{u_i, i = 1,\ldots,n\}]$,
  where, $u_i(s,\theta) = v_i[s,\theta] + C_i(s)$
- Consider, C_i^{II} defined by,

$$C_i^{II}(s) = \sum_{j \neq i} E[v_j[s^*,\theta_j] | s_0^* = s_0] - A_i$$

A_i is any constant.

Groves Result

$U^{II} = \{u_i^{II}, i = 1, \ldots, n\}$ where,
$u_i^{II}(s, \theta) = v_i[s, \theta_i] + C_i^{II}(s)$

Theorem

Given the organization model $T = [N, \Theta, \{S_i, i = 0, \ldots, n\}, u_0]$ with the conglomerate specifications, and if T satisfies Assumption A, then U^{II} is an optimal incentive structure in the class $\mathscr{I}$.

Groves Mechanism

- Consider a set of possible alternatives, $\mathscr{K}$, and agents with quasi-linear utility functions, such that

$$u_i(k; t_i; \theta_i) = v_i(k; \theta_i) - t_i$$

$$\text{Let, } k^*(\hat{\theta}) = \text{argmax}_{k \in \mathscr{K}} \sum_i v_i(k; \hat{\theta}_i)$$

  The payment rule in a Groves mechanism is defined as:

$$t_i(\hat{\theta}) = h_i(\hat{\theta_{-i}}) - \sum_{j \neq i} v_j(k^*; \hat{\theta}_j)$$

- Social Choice Function $f() = (k, t_0, t_1, \ldots, t_n)$
- Groves Theorem :
  A social choice function $f()$ which is allocatively efficient is *DSIC* if payments are according defined as above.

Some Important results

- Gibbard-Satterwaite : A ex-post efficient SCF which can be implemented in DS is dictatorial.
- Green-Laffont[3] : A direct revelation Mechanism which is DSIC, is necessarily Groves Mechanism.
- Green-Laffont[3] : In quasi linear environment, it is impossible to have a DSIC social choice function which is allocatively efficient and strict budget balanced at the same time.

Scenario

- A group of n agents must assign p identical objects, where $p < n$. We have a simple rationing problem: each agent claims a unit but all claims can't be met.
- Assume preferences are quasi linear in money.
- We can use Groves Mechanism (VCG Mechanisms).
- But VCG Mechanisms are not budget balanced.

- Laffont and Maskin (1979)[5] : Redistribute the surplus among participating agents.
- Carvollo(2006)[6] : Proposed rebate functions that depends only on $(p+2)$ highest bids.
- Guo and Contizer(2006)[7][1] : Defined performance ratio of a mechanism as,

$$\max_{\theta \in \Theta} \frac{\text{Budget Surplus}}{\text{Revenue to Aucioneer}}$$

- Herve Moulin(2007)[4] : Defined Notion of Efficiency Loss.

$$L(n,p) = \max_{\theta \in \Theta} \frac{\text{Budegt Surpls}}{\text{Efficient Surplus}}$$

[1] Un-dominated VCG redistribution Mechanism.

- Given a profile of valuations $a \in \mathbb{R}_+^N$, the vector $a^* \in \mathbb{R}_+^N$ is its permutation where coordinates are arranged decreasingly. i.e. $a^{*1} \geq a^{*2} \geq \ldots a^{*n}$
- Efficient Surplus : $v_p(a) = a^{*1} + \ldots + a^{*p}$
- $u_i(a) = v_p(a) - h_i(a_i) \text{for all } a \in \mathbb{R}_+^N$
- Budget Imbalance : $\Delta(a) = v_p(a) - \sum_{i \in N} u_i(a)$
- $$L(n,p) = \max_{a \in \mathbb{R}_+^N \setminus \{0\}} \frac{|\Delta(a)|}{v_p(a)}$$

- *Feasibility* $(F) : \Delta(a) \geq 0$.
- Voluntary Participation (VP) : $u_i(a) \geq 0 \;\; \forall\, a \, \forall\, i$
  $\Leftrightarrow h_i(a_{-i}) \leq v_p(a_{-i}) \;\; \forall\, a_{-i} \, \forall\, i$
- Note : F and VP $\Rightarrow 0 \leq L(n, p) \leq 1$
- In Vickrey Auction : $h_i^{vick}(a_{-i}) = v_p(a_{-i})$
- Consider profile 'a' such that $a^{*1} = \ldots = a^{*(p+1)}$. $\Rightarrow L^{vick}(n, p) = 1$
- Moulin : write $h_i(a_{-i})$ as $h_i(a_{-i}) = v_p(a_{-i}) - r_p(i, a_{-i})$ where $r_p(i, a_{-i})$ is rebate function.

Results

Define,

$$B_s^{t,u} = \sum_{k=t}^{u} \left(\begin{array}{c} s \\ k \end{array} \right). \; B_s^{t\rightarrow} = B_s^{t,s} \; B_s^{\rightarrow t} = B_s^{0,t}$$

- Under F and VP, the smallest efficiency loss is given by,

$$L^*(n,p) = \frac{\left(\begin{array}{c} n-1 \\ p \end{array} \right)}{B_{n-1}^{p\rightarrow}}$$

The following linear rebate functions define an optimal mechanism

$$r_p^*(a_i) = \sum_{k=p+1}^{n-1} (-1)^{k-p-1} \frac{pL^*(n,p)}{kL^*(n,k)} a_{-i}^{*k} \text{ if } p \leq n-2; \; r_{n-1}(a_{-i}) = 0$$

Results continued

- Under F, the smallest efficiency loss, $\hat{L}(n,p)$ is given by,

$$\hat{L}(n,1) = L^*(n,1); \hat{L}(n,p) = \frac{\begin{pmatrix} n-1 \\ p \end{pmatrix}}{B_{n-1}^{p\rightarrow} + \frac{n}{p} B_{n-2}^{\rightarrow(p-2)}}$$

- Among all VCG mechanisms, the smallest cost index $L^{\#}(n,p)$ is such that,

$$\frac{1}{2+\frac{1}{n-1}}\hat{L}(n,p) \leq L^{\#}(n,p) \leq \frac{1}{2}\hat{L}(n,p)$$

Asymptotic efficiency

1. $L^*(n,p)$ increases strictly in p. Decreases strictly in n.
2. $\hat{L}(n,p)$ increases in n for $p \leq n \leq 2p-1$, $\hat{L}(n,p)$ decreases in n if $2p \leq n$
3. $\hat{L}(n,p)$ increases in p for $1 \leq p \leq \{\frac{n}{2}\}$ decreases in p if $\{\frac{n}{2}\} \leq p \leq n$
4. Loosely speaking, $\hat{L}(n,p)$ and $L^*(n,p)$ converges exponentially fast to zero in n if $\frac{p}{n} < \frac{1}{2}$ and as $\frac{1}{\sqrt{n}}$ if $\frac{p}{n} \simeq \frac{1}{2}$.
5. If $\frac{p}{n} > \frac{1}{2}$, un-voluntary mechanisms still allow exponentially fast efficiency while, V ones preclude asymptotic efficiency altogether.

Open problems

- Non - VCG Mechanisms.
- Heterogenous Objects.

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Thank You!!!