

Almost Budget Balanced VCG Mechanisms

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Part I

- VCG Mechanism
 - Groves : Incentive in Teams.
 - Groves Mechanism.
 - Some Possibility and Impossibility Results.

Part II

- Almost Budget Balanced VCG Mechanisms
 - Scenario Under consideration.
 - Efficiency Loss.
 - Optimal Mechanisms.

- Open Problems

Consider a conglomerate-type organization consisting of a set of semi-autonomous subunits that are coordinated by the organization's head. The head's incentive problem is to choose a set of employee compensation rules that will induce his subunit managers to communicate accurate information and take optimal decisions. (T Groves [1])

N = The set of agents $\{0, 1, 2, 3, \dots, i, \dots, n\}$
 0 : The head.

$1, 2, \dots, n$: Subunit Managers

Θ_i = Type profile of an agent i

θ_i = Type of agent i

Θ = $\prod_{i=0}^{i=n} \Theta_i$

θ = $(\theta_0, \dots, \theta_i, \dots, \theta_n)$

S_i = Strategy Space of the agent i

S = $\prod_{i=0}^{i=n} S_i$

s = $(s_0, \dots, s_i, \dots, s_n)$

u_i	:	$S \times \Theta \rightarrow \mathbb{R}$ The Pay off function to agent i .
T	=	$\{N, \Theta, \{S_i : i \in N\}, u_0\}$ The Team Model.
G	=	$\{N, \Theta, \{S_i : i \in N\}, \{u_i : i \in N\}\}$ The Induced Game .
(\hat{s}/s_i)	=	$(\hat{s}_0, \dots, s_{i-1}, s_i, s_{i+1}, \dots, \hat{s}_n)$

Table: Notation Incentive in Teams Problems

The Team's Problem

The team's objective is assumed to be to choose a joint strategy $s^* \in S$, if one exists, that maximizes the expected value of the team payoff function u_0 .

$$E[u_0(s^*)] \equiv \max_{s \in S} E[u_0(s)]$$

Assumption A

There exists a $s^ \in S$ such that*

- i. $E[u_0(s^*)] \geq E[u_0(s)] \quad \forall s \in S$
- ii. $E[u_0(s^*)] > E[u_0(s^*/s_i)] \quad \forall s_i \neq s_i^*, s_i \in S_i \quad \forall i \in N$

Incentive Problem

- A set $U = \{u_i : i = 1, \dots, n\}$ of employee payoff functions is called an *incentive structure*, and an incentive structure $U^* = \{u_i^*, i = 1, \dots, n\}$ is called *optimal* if, for the joint strategy s^* satisfying Assumption,

$$E[u_i^*(s^*)] = \max_{s_i \in S_i} E[u_i^*(s^*/s_i)] \text{ uniquely for all } i = 1, \dots, n.$$

- The *incentive problem* of the organization head is then to find an optimal incentive structure U^* , or equivalently, since every incentive structure U defines an $(n + 1)$ -person game, to choose the optimal game for his organization to play.

Incentive Structure : Two Common Systems

① *'Paid worker incentive structure.'*

Define, $U^0 = \{u_i^0, i = 1, \dots, n\}$ by,

$$u_i^0(s, \theta) = \begin{cases} 1 & : \text{ if } s_i = s_i^* \\ 0 & : \text{ otherwise} \end{cases}$$

Disadvantage : Assumes complete information setting.

② Define, $U^I = \{u_i^I, i = 1, \dots, n\}$ by, ($i = 1, \dots, n$)

$$u_i^I(s, \theta) = \alpha_i * u_0(s, \theta) + A_i$$

where, α_i is positive constant and A_i any constant.

This is called *'profit sharing incentive structure'*.

Disadvantage : Does not discriminate between optimally and non-optimally performing employees.

Specifications

1

$$u_0(s, \theta) = \sum_{i=1}^n v_i[s, \theta_i] + v_0[s, \theta_0]$$

2 *The portion $v_i[s, \theta_i]$ of the organization payoff accrues directly to the i^{th} subunit.*

- $\mathcal{I} = [U = \{u_i, i = 1, \dots, n\}]$,
where, $u_i(s, \theta) = v_i[s, \theta] + C_i(s)$
- Consider, C_i'' defined by,

$$C_i''(s) = \sum_{j \neq i} E[v_j[s^*, \theta_j] | s_0^* = s_0] - A_i$$

A_i is any constant.

$$U^{\parallel} = \{u_i^{\parallel}, i = 1, \dots, n\} \text{ where,}$$
$$u_i^{\parallel}(s, \theta) = v_i[s, \theta_i] + C_i^{\parallel}(s)$$

Theorem

Given the organization model $T = [N, \Theta, \{S_i, i = 0, \dots, n\}, u_0]$ with the conglomerate specifications, and if T satisfies Assumption A, then U^{\parallel} is an optimal incentive structure in the class \mathcal{I} .

- Consider a set of possible alternatives, \mathcal{K} , and agents with quasi-linear utility functions, such that

$$u_i(k; t_i; \theta_i) = v_i(k; \theta_i) - t_i$$

$$\text{Let, } k^*(\hat{\theta}) = \operatorname{argmax}_{k \in \mathcal{K}} \sum_i v_i(k; \hat{\theta}_i)$$

The payment rule in a Groves mechanism is defined as:

$$t_i(\hat{\theta}) = h_i(\hat{\theta}_{-i}) - \sum_{j \neq i} v_j(k^*; \hat{\theta}_j)$$

- Social Choice Function $f() = (k, t_0, t_1, \dots, t_n)$

- Groves Theorem :

A social choice function $f()$ which is allocatively efficient is *DSIC* if payments are according defined as above.

Some Important results

- Gibbard-Satterwaite : A ex-post efficient SCF which can be implemented in DS is dictatorial.
- Green-Laffont[3] : A direct revelation Mechanism which is DSIC, is necessarily Groves Mechanism.
- Green-Laffont[3] : In quasi linear environment, it is impossible to have a DSIC social choice function which is allocatively efficient and strict budget balanced at the same time.

- A group of n agents must assign p identical objects, where $p < n$. We have a simple rationing problem: each agent claims a unit but all claims can't be met.
- Assume preferences are quasi linear in money.
- We can use Groves Mechanism (VCG Mechanisms).
- But VCG Mechanisms are not budget balanced.

- Laffont and Maskin (1979)[5] : Redistribute the surplus among participating agents.
- Carvollo(2006)[6] : Proposed rebate functions that depends only on $(p + 2)$ highest bids.
- Guo and Contizer(2006)[7]¹ : Defined performance ratio of a mechanism as,

$$\max_{\theta \in \Theta} \frac{\text{Budget Surplus}}{\text{Revenue to Auctioneer}}$$

- Herve Moulin(2007)[4] : Defined Notion of Efficiency Loss.

$$L(n, p) = \max_{\theta \in \Theta} \frac{\text{Budegt Surpls}}{\text{Efficient Surplus}}$$

¹Un-dominated VCG redistribution Mechanism.

- Given a profile of valuations $a \in \mathbb{R}_+^N$, the vector $a^* \in \mathbb{R}_+^N$ is its permutation where coordinates are arranged decreasingly. i.e. $a^{*1} \geq a^{*2} \geq \dots a^{*n}$
- Efficient Surplus : $v_p(a) = a^{*1} + \dots + a^{*p}$
- $u_i(a) = v_p(a) - h_i(a_i)$ for all $a \in \mathbb{R}_+^N$
- Budget Imbalance : $\Delta(a) = v_p(a) - \sum_{i \in N} u_i(a)$
-

$$L(n, p) = \max_{a \in \mathbb{R}_+^N \setminus \{0\}} \frac{|\Delta(a)|}{v_p(a)}$$

- *Feasibility (F)* : $\Delta(a) \geq 0$.
- *Voluntary Participation (VP)* : $u_i(a) \geq 0 \quad \forall a \quad \forall i$
 $\Leftrightarrow h_i(a_{-i}) \leq v_p(a_{-i}) \quad \forall a_{-i} \quad \forall i$
- Note : F and VP $\Rightarrow 0 \leq L(n, p) \leq 1$
- In Vickrey Auction : $h_i^{vick}(a_{-i}) = v_p(a_{-i})$
- Consider profile 'a' such that $a^{*1} = \dots = a^{*(p+1)}$. $\Rightarrow L^{vick}(n, p) = 1$
- Moulin : write $h_i(a_{-i})$ as $h_i(a_{-i}) = v_p(a_{-i}) - r_p(i, a_{-i})$ where $r_p(i, a_{-i})$ is **rebate function**.

Define,

$$B_s^{t,u} = \sum_{k=t}^u \binom{s}{k} \cdot B_s^{t \rightarrow} = B_s^{t,s} \quad B_s^{\rightarrow t} = B_s^{0,t}$$

- Under F and VP, the smallest efficiency loss is given by,

$$L^*(n, p) = \frac{\binom{n-1}{p}}{B_{n-1}^{p \rightarrow}}$$

The following linear rebate functions define an optimal mechanism

$$r_p^*(a_i) = \sum_{k=p+1}^{n-1} (-1)^{k-p-1} \frac{pL^*(n, p)}{kL^*(n, k)} a_{-i}^{*k} \text{ if } p \leq n-2; \quad r_{n-1}(a_{-i}) = 0$$

- Under F, the smallest efficiency loss, $\hat{L}(n, p)$ is given by,

$$\hat{L}(n, 1) = L^*(n, 1); \hat{L}(n, p) = \frac{\binom{n-1}{p}}{B_{n-1}^{p \rightarrow} + \frac{n}{p} B_{n-2}^{(p-2) \rightarrow}}$$

- Among all VCG mechanisms, the smallest cost index $L^\#(n, p)$ is such that,

$$\frac{1}{2 + \frac{1}{n-1}} \hat{L}(n, p) \leq L^\#(n, p) \leq \frac{1}{2} \hat{L}(n, p)$$

Asymptotic efficiency

- 1 $L^*(n, p)$ increases strictly in p . Decreases strictly in n .
- 2 $\hat{L}(n, p)$ increases in n for $p \leq n \leq 2p - 1$, $\hat{L}(n, p)$ decreases in n if $2p \leq n$
- 3 $\hat{L}(n, p)$ increases in p for $1 \leq p \leq \{\frac{n}{2}\}$ decreases in p if $\{\frac{n}{2}\} \leq p \leq n$
- 4 Loosely speaking, $\hat{L}(n, p)$ and $L^*(n, p)$ converges exponentially fast to zero in n if $\frac{p}{n} < \frac{1}{2}$ and as $\frac{1}{\sqrt{n}}$ if $\frac{p}{n} \simeq \frac{1}{2}$.
- 5 If $\frac{p}{n} > \frac{1}{2}$, un-voluntary mechanisms still allow exponentially fast efficiency while, V ones preclude asymptotic efficiency altogether.

- Non - VCG Mechanisms.
- Heterogenous Objects.

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Thank You!!!