Games With Incomplete Information
A Nobel Lecture by John Harsanyi

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Course: Topics in Game Theory
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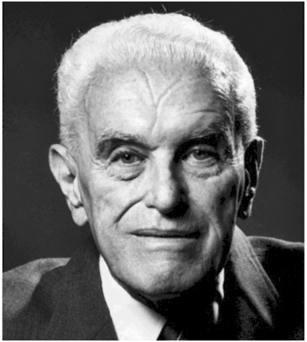
Agenda

- Biography of John C Harsanyi
- Contributions
- Nobel Lecture
  - C-Games and I-Games
  - Two Person I-Games
  - Converting I-Games to C-Games

Biography of John Harsanyi

- Name: John C Harsanyi
- Date of Birth: May 29, 1920
- Place: Budapest, Hungary
- School: Lutheran Gymnasium in Budapest
- First Prize in Mathematics at the Hungary-wide annual competition for high school students
- Graduated in 1937
- Opted pharmacy in accordance with his parents' wishes



Journey

- 1944: Harsanyi was drafted into a forced-labor unit near Budapest. He managed to escape narrowly
- 1947: Harsanyi earned a PhD in philosophy at the University in Budapest where he joined as Assistant Professor
- 1948: Stalinist regime seized power in Hungary. He had to resign from the university and return to work in his father's pharmacy
- 1950: Harsanyi and his soon-to-be wife, Anne, escaped across the border to Austria, and emigrated to Australia
- 1951: He got married

- Harsanyi worked in factories during the day while earning an MA in economics at the University of Sydney at night
- 1954: he was appointed lecturer in economics at the University of Queensland in Brisbane
- 1956: He enrolled in the PhD program in economics at Stanford University, writing his dissertation on game theory under the guidance of the future Nobel Laureate Kenneth Arrow
- 1964: Joined University of California, Berkeley (Hass School of Business)
- 1994: Was awarded Nobel memorial prize in economics
- Harsanyi was awarded seven honorary doctorates by universities around the world.
- 2000: August 9, he is no more. Was suffering from Alzheimer.

Major Contributions

- Harsanyi is well known for contributing to the study of equilibrium selection
- John C. Harsanyi, "A simplified bargaining model for the n-person cooperative game," International Economic Review 4 (1963), 194-220.
- Shapley Value: required payoffs with transferable utilities.
- Nash Bargaining solution: works only for two players.
- He showed how the Nash bargaining solution and the Shapley value could be unified into one general solution concept that could be applied to any cooperative game with complete information
- He wrote four books.

Seminal Contributions

- The monumental three-part series of papers on games with incomplete information, which John Harsanyi published in 1967 and 1968 led to theory of Bayesian games.
- Defined Bayesian game model to model uncertainty about a game by bringing the uncertainty into the game model itself
- John C. Harsanyi "Games with incomplete information played by Bayesian players," Management Science 14 (1967-1968), 159-182, 320-334, 486-502.

Nobel Lecture by John Harsanyi

- Game theory is a theory of *strategic interaction*
- Classical economic theory did manage to sidestep the game-theoretic aspects of economic behavior by postulating perfect competition
- However, perfect competition is an unrealistic assumption.
- In such case, game theory is definitely an important analytical tool in understanding the operation of economic system.

C-Games and I-Games

- 1965 - 69, the U.S. Arms Control and Disarmament Agency employed a group of about ten young game theorists as consultants, of which Harsanyi was a member
- He realized: each side is poorly informed about the other side's position in terms of variables of negotiations
- This led to distinguish between games with complete information and games with incomplete information
- Games with *complete information*: each player has complete information about the games
- Games with *incomplete information*: at least some of them, lack full information about the basic mathematical structure of the game
- Game with complete information are referred to as *C-Games* and with incomplete information are referred to as *I-Games*.

Two person I-Games

A model based on higher and higher-order expectations

- Two players who do not know each other's payoff functions.
- u_1 and u_2 be the player 1 and 2's payoff functions.
- Natural way to deal with this is,
  Player 1 will take expectation of u_2, $E_1[u_2]$ and player 2 will do same, $E_2[u_1]$, before deciding their strategies.
- These are called first order expectations.
- Then player 1 will take second order expectation of 2's first-order expectation $E_2[u_1]$, that is, $E_1[E_2[u_1]]$ and same for player 2.
- And higher and higher order expectations are taken.
- This model is complicated and becomes more complicated for n-person I-Games.

Notation

θ_1	Type of player 1, $= \theta_1^1, \theta_1^2, \ldots, \theta_1^k, \ldots, \theta_1^K$
θ_2	Type of player 2 $= \theta_2^1, \theta_2^2, \ldots, \theta_2^m, \ldots, \theta_1^M$
s_i^j	Strategy played by player i wen his type is θ_i^j.
s_i	$= (s_i^1, s_i^2, \ldots, s_i^j, \ldots)$
$Pr(\theta_1^k, \theta_2^m)$	Probability that player is of the type θ_1^k and the player 2 is of that of type θ_2^m. $= p_{km}$
$u_i(\theta_1, \theta_2, s_1, s_2)$	Player i's payoff when types are θ_1, θ_2 and strategies played by players are s_1 and s_2.
$U_i(\theta_i, s_1, s_2)$	Expected utility to player i, when his type is θ_i and strategies played by players are s_1 and s_2.

Table: Notation

Harsanyi's Model

- To model the uncertainty of the player 2 about the true nature of the that of player 1, assume that there are K different possible types of the player 1, to be called types .
- Similarly, there are M different possible types of the player 2.
- C-games: player centered.
- I-Games can be player centered or type centered

Type centered interpretation of I-Game

- In this interpretation, each type is a player in the game.
- Suppose, player 1 is of type θ_1^k and player 2 is of type θ_2^m.
- θ_1^k and θ_2^m are called *active types*
- Rest of the types are *inactive types*
- Instead of payoff and strategy of player 1, in this interpretation it is described as payoff and strategy of type player θ_1^k

Converting I-Game to C-Game

In an I-Game G,

1. θ_1^k, θ_2^m are established facts from the very begining of the game, and they are not facts brought about by some move(s) made during the game. Consequently, these two facts must be considered to be parts of the basic mathematical structure of this game G.
2. Player 1 will know θ_1^k but not θ_2^m.
3. Player 2 will know θ_2^m but not θ_1^k.

When G is converted to C-Game G^*,

- Statements 2 and 3 above will be still valid.
- Statement 1 undergoes radical change.
  these two types will now become the results of a chance move made by lottery L during the game.
- Thus the I-Game G becomes C-Game G^*, but with imperfect information.

Type centered interpretation of G^*

- Define,

$$\pi_1^k(m) = \frac{p_{km}}{\sum_{j=1}^{K} p_{jm}}$$

- Payoff to type player θ_i^k,

$$u_{\theta_1^k} = U_1(\theta_1^k, s_1, s_2) = \sum_{m=1}^{M} \pi_1^k(m) u_1(\theta_1^k, \theta_2^m, s_1, s_2)$$

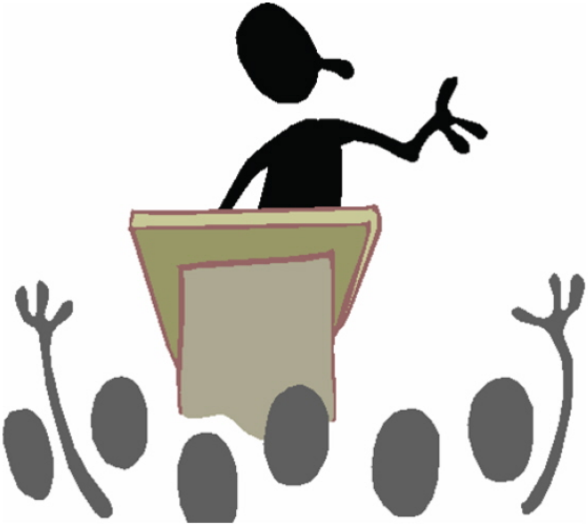
- Similarly for player 2.
- Thus we have $K + M$ player C-Game.

Note: If any I-Game is converted properly to C-game, player centered interpretation and type centered interpretation gives the same analysis of the original game.

Some of Harsanyi's important publications

- John C. Harsanyi, "Approaches to the bargaining problem before and after the theory of games," Econometrica 24 (1956) 144-157.
- John C. Harsanyi, "Theoretical analysis in social science and the model of rational behavior," Australian Journal of Politics and History 7 (1961a), 60-74.
- John C. Harsanyi, "On the rationality postulates underlying the theory of cooperative games," Journal of Conflict Resolution 5 (1961b), 179-196
- John C. Harsanyi, "A simplified bargaining model for the n-person cooperative game," International Economic Review 4 (1963), 194-220.
- John C. Harsanyi "Games with incomplete information played by Bayesian players," Management Science 14 (1967-1968), 159-182, 320-334, 486-502.
- John C. Harsanyi and Reinhard Selten, A General Theory of Equilibrium Selection in Games, MIT Press (1988).

Questions?



Thank You!!!