Dynamic Matchings with a Fall-back Option

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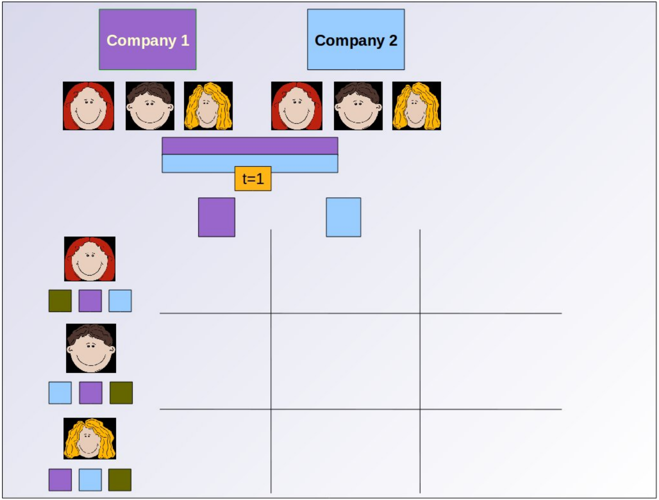
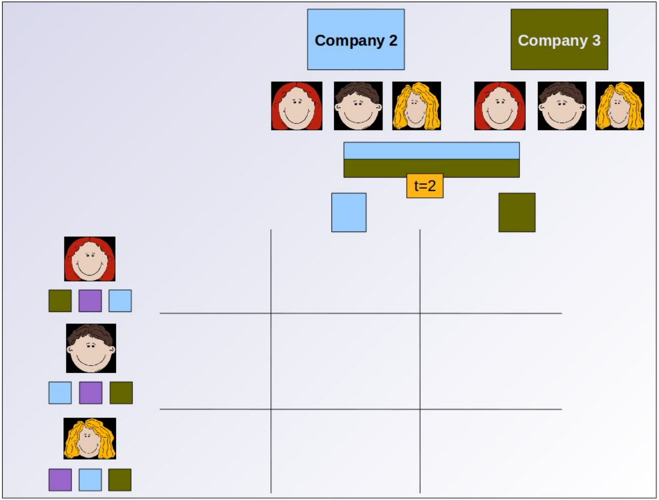
Agenda

- Introduction
- Related work
- Our Approach
- Summary

Motivating Example



Motivating Example



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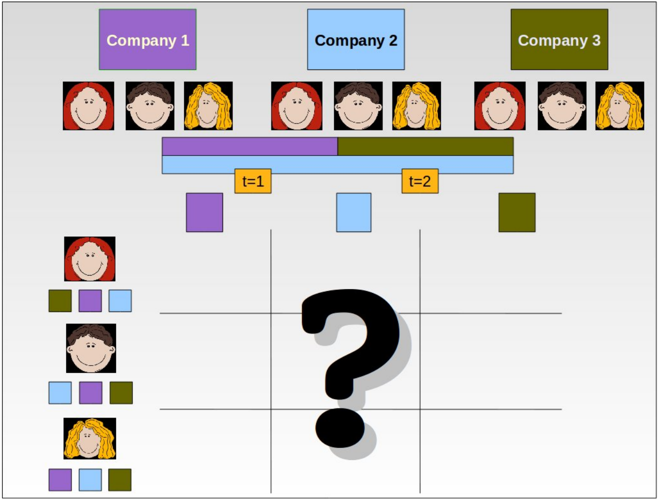


Observations

- This can be considered as a matching problem
- Students are static (= Men, M)
  - The preferences of the students are private
- Companies arrive-depart dynamically (= Women, W)
- We can assume that a company won't lie about its preferences
  - Typically it would be known what grades, skill-sets are required for a particular position in the company
- We focus on incentive properties on static side

Desirable Properties

- Blocking pair: (m, w) blocks the matching if they prefer to match with each other than their current match
- Stability: no blocking pair
- Strategyproof: For each agent, for any arrival-departure schedule, for any preferences, it is a best response to report preferences truthfully
- Good Rank Efficiency. Lower the $Rank(f)$, better[1]

[1] Average rank assigned by the agents to their match

Male-Proposal Deferred Acceptance

Gale-Shapley[2] proposed,

- Each man proposes to his most preferred woman
- Each woman keeps her most preferred man among the proposals received and rejects all the others
- The men who are rejected in the above step propose to their next preferred woman
- If a woman receives a match better than her current match, she is matched with the new man and the previous man is not matched
- The process continues till all the men are matched

[2] D. Gale and L. S. Shapley, "College admissions and the stability of marriage", The American Mathematical Monthly, 69(1), 9-15, (January 1962)

Important Static Case Results

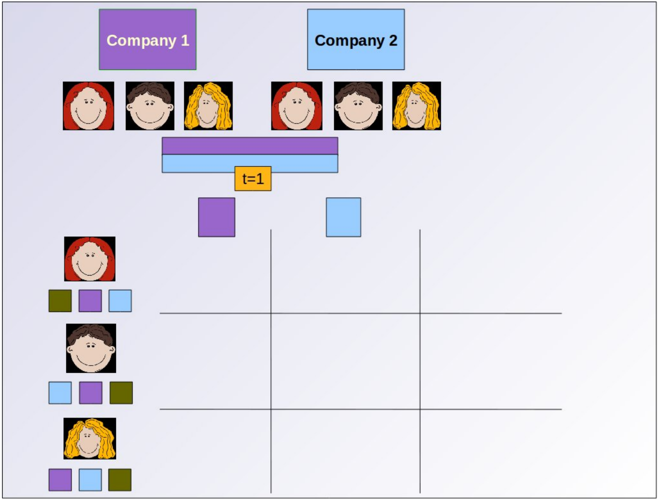
- Deferred acceptance is stable
- Male-Proposal Deferred acceptance is strategyproof for men[3]
- No stable algorithm is strategyproof

[3] A. E. Roth, "The economics of matching: Stability and incentives", Mathematics of Operations Research, 7(4), 617-628, (1982).

On-line Deferred Acceptance

1. In each period, run Male-Proposal Deferred Acceptance on current population. All these matches are temporary
2. If any woman is departing in a period, the man with whom she is currently matched is made final

But It Fails to Be Strategyproof



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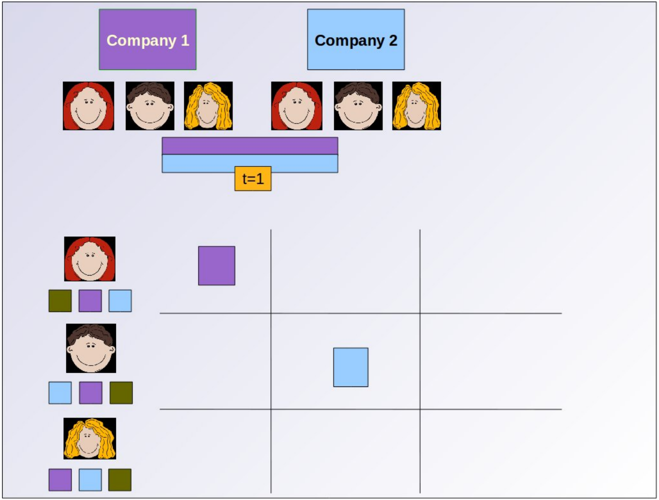

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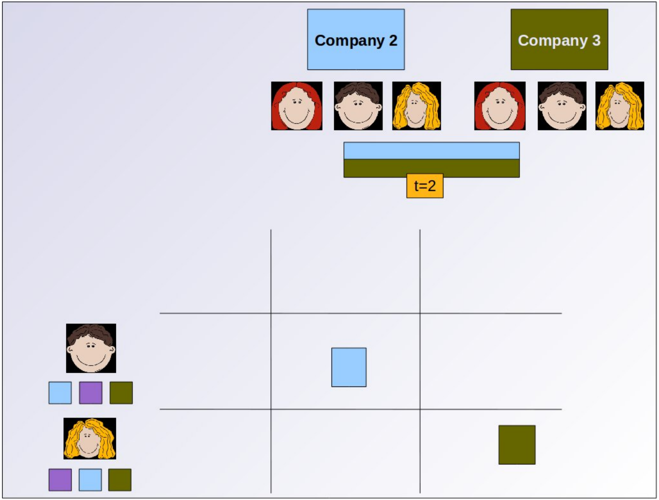

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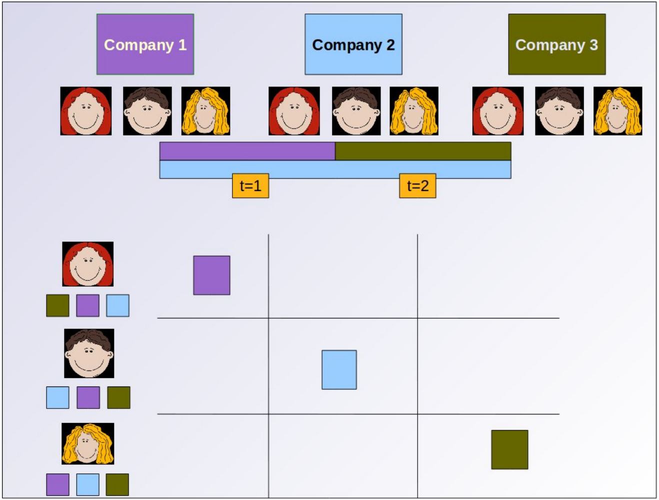

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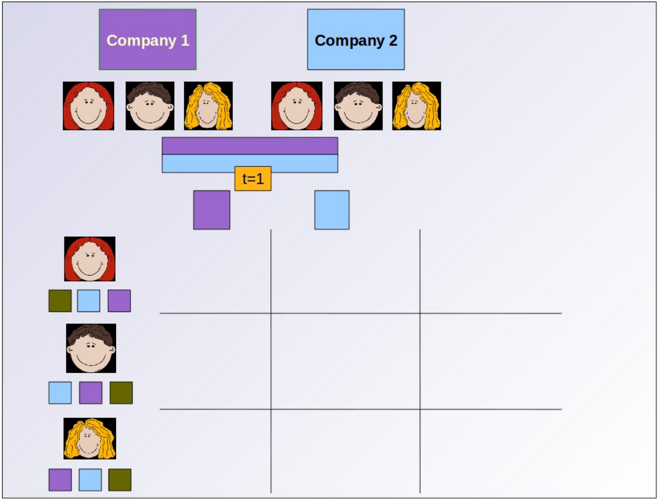

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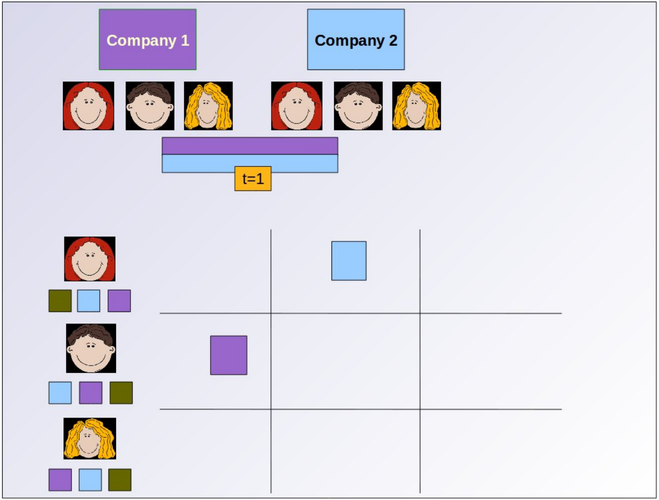

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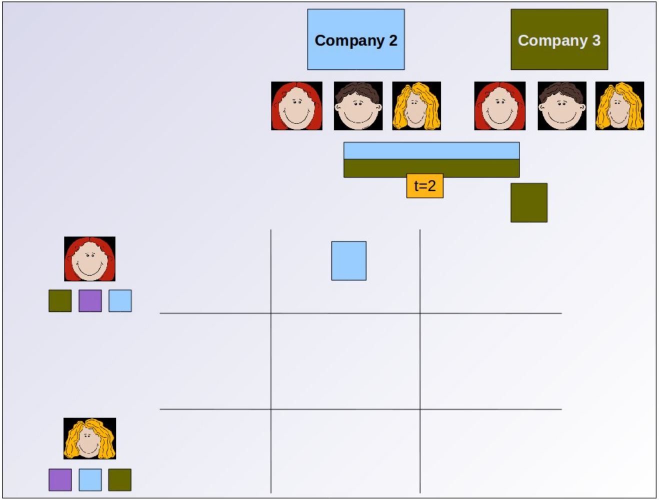

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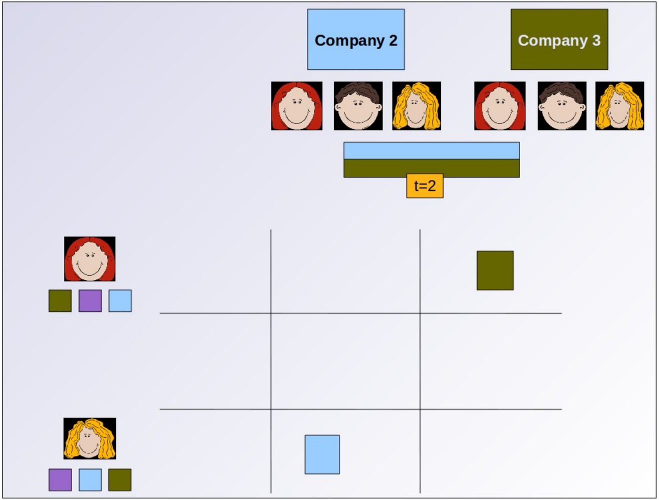

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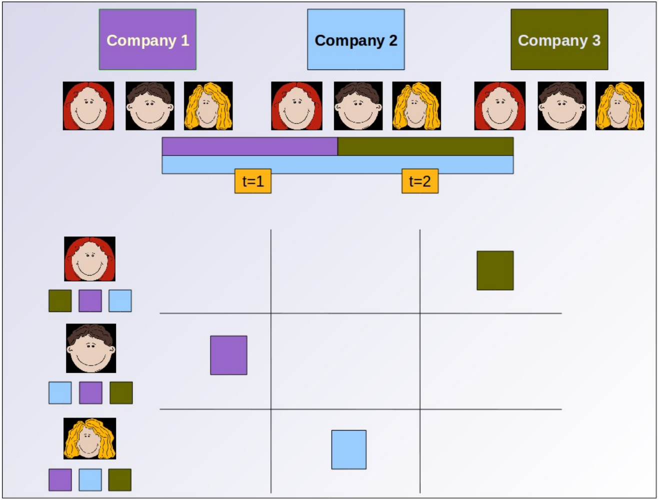
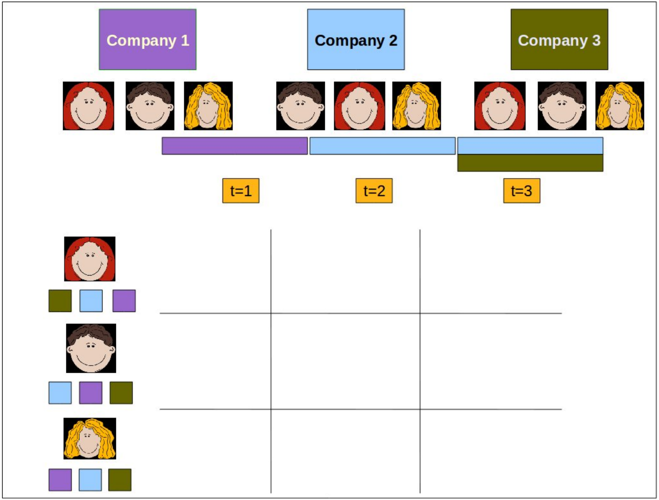


We propose: GSODAS

- Why does the On-line Deferred Acceptance fail?
- What if a man has option to decommit from a match if he gets a better match later?
- **G**eneralized **S**table, **O**n-line **D**eferred **A**cceptance with **S**ubstitutes
- Algorithm:
  - Run Male-Proposal Deferred Acceptance in each period.
  - If a man receives a better match, he can decomit from his current match
    - If this women has already left, she receives a substitute for him
  - Match a departing women with her current match

GSODAS: An Example



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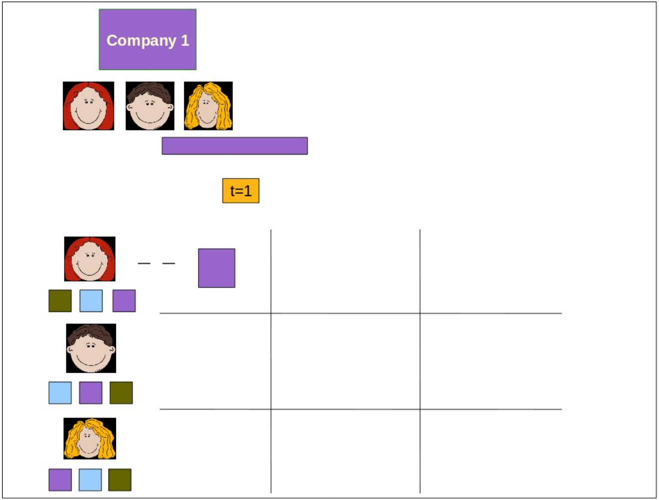

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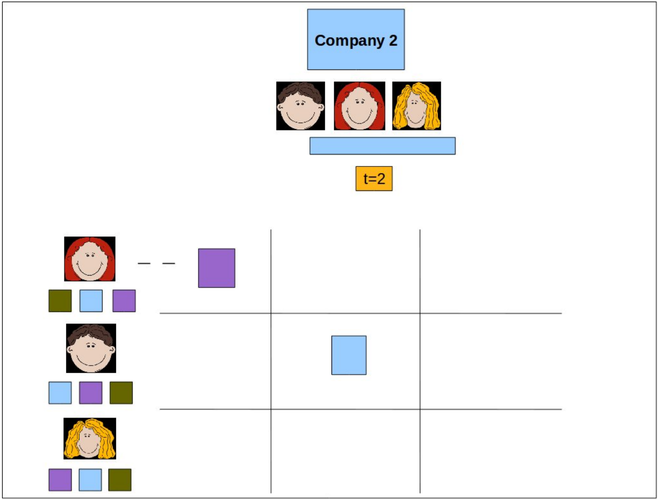

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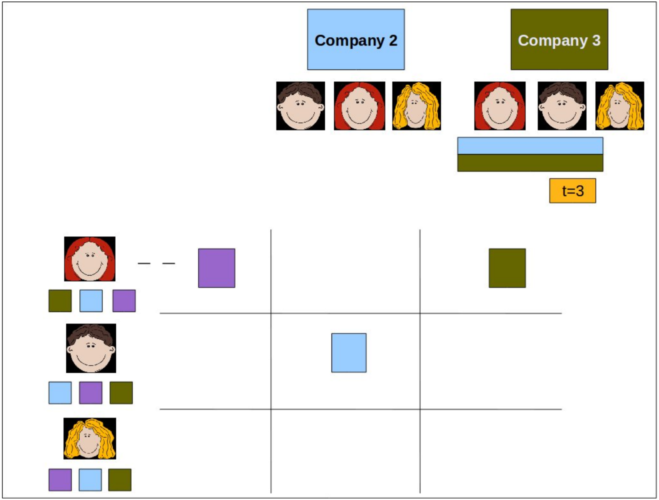

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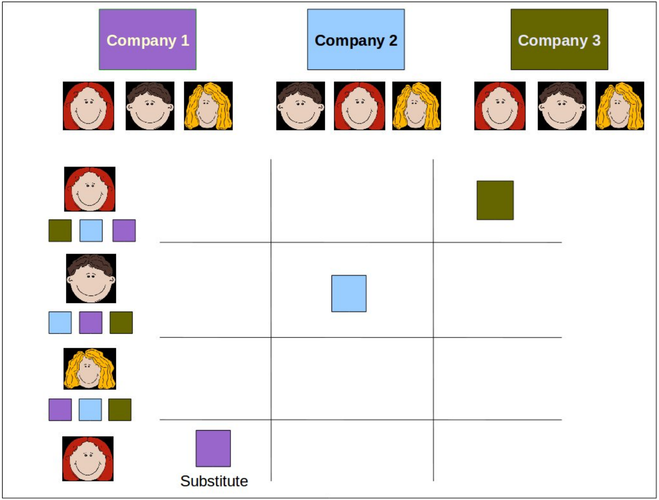


Properties of GSODAS

- Stable
- Strategyproof for men
- What about # substitutes required?

Worst Case Optimality of GSODAS

- GSODAS achieves stability at the cost of substitutes
- There is a necessary trade-off between usage of substitutes and stability
- Let $n = \alpha T,\ \alpha \in \{1, 2, 3, \ldots\}$, where T is number of rounds in the game
- For matching μ use the metric

  $\boxed{S(\mu) = \#\text{ Unstable men in } \mu + \#\text{ Substitutes used in } \mu}$
- We show,

> Any on-line algorithm for matching, in worst case analysis, $S(\mu) \geq \alpha(T - 1)$ and for GSODAS $S(\mu) \leq \alpha(T - 1)$ with equality in the worst case.

- This implies, **GSODAS achieves stability optimally**

Other Strategyproof On-line Algorithm

- Strategyproof Randomized On-line Matching Algorithm
- ROMA1
  - Run Male-Proposal Deferred Acceptance on departing women and men chosen at random.
  - This match is final
- ROMA2
  - In each period, if # women at least τ, run Male-Proposal Deferred Acceptance on all women and men chosen at random.
  - This match is final

Benchmark: CONSENSUS (Non-Strategyproof)

- On-line stochastic optimization[4]
- In each period, simulate the future arrival of the women by sampling future possible scenarios
- For each present woman, find out which man is getting matched most frequently. Run Male-Proposal Deferred Acceptance with present women and men who are most often matched.
- Commit only those matches that involve departing women

[4] P. Van Hentenryck and R. Bent, Online Stochastic Combinatorial Optimization, MIT Press, 2006.

Number of Substitutes required by GSODAS

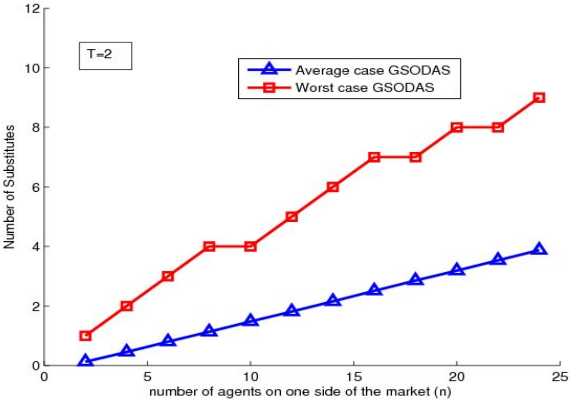


Figure: The number of substitutes required for men in GSODAS as n increases, fixing $T = 2$.

Number of Substitutes required by GSODAS

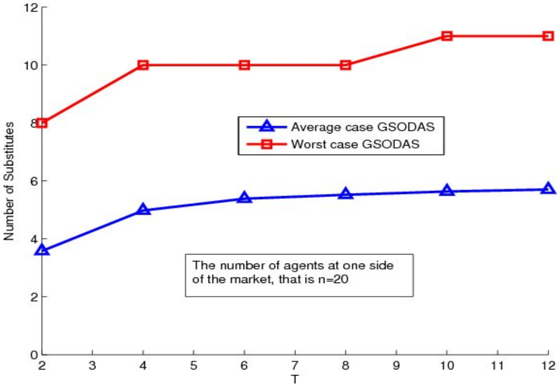


Figure: The number of substitutes required for men in GSODAS as T increases, fixing $n = 20$.

Stability vs Rank Efficiency

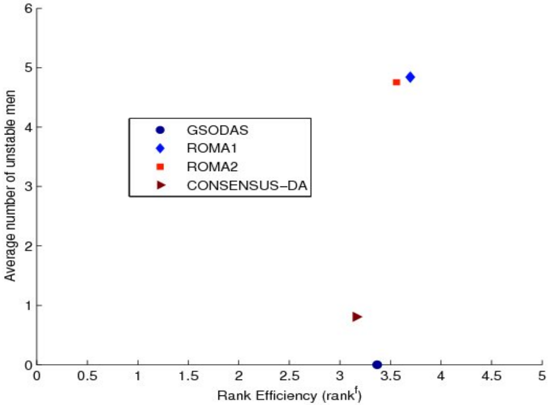


Figure: The rank-efficiency (x-axis) vs. the number of unstable men (y-axis) for $n = 10$ and $T = 2$.

Stability vs Rank Efficiency

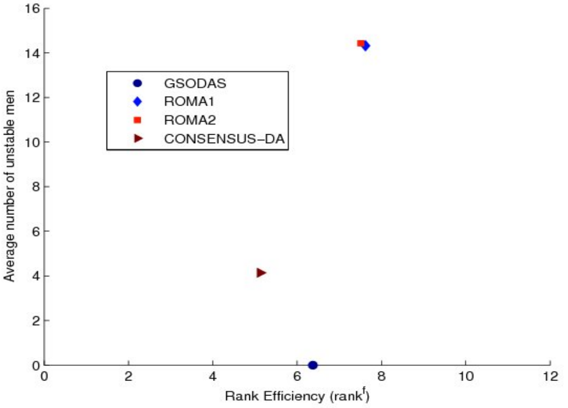


Figure: The rank-efficiency (x-axis) vs. the number of unstable men (y-axis) for $n = 20$ and $T = 4$.

Conclusion

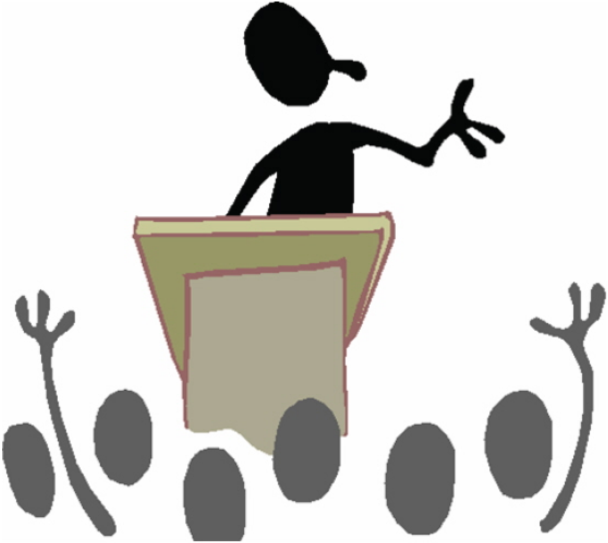
- The naive deferred acceptance fails in on-line settings
- We introduce the fall-back option
- GSODAS is stable and strategyproof (for men) at the cost of substitutes
- GSODAS achieves stability with minimal number of substitutes (worst case analysis)
- Experiments show GSODAS requires significantly lower substitutes than the worst case bounds
- GSODAS performs quite well for rank efficiency

Future Work

- In experiments: as time horizon (T) increase as many as 55% men use substitutes in the worst case
- Likely to be unacceptable in many practical situations

$\Rightarrow$ Relax the stability notion.

Questions?



Thank You!!!

Proof of Claim 1

Yes this one and next one are extra slides.

- For GSODAS, # Unstable men $= 0$ and # Substitutes $\leq \alpha(T-1)$
- Construct preference profile for which for any on-line matching algorithm, $S(\mu) = \alpha(T-1)$

Preferences

m_1	$(T, \ldots, 2, 1, w)$	w_1	$(1, 2, \ldots, T, m), \ a = d = 1$
m_2	$(T, \ldots, 2, 1, w)$	w_2	$(\mu(w_1), 1, 2, \ldots, T, m), \ a = d = 2$
$\vdots$			
m_T	$(T, \ldots, 2, 1, w)$	w_T	$(\mu(w_{T-1}), 1, 2, \ldots, T, m), \ a = d = T$
m_{T+1}	$(2T, \ldots, T+2, T+1, w)$	w_{T+1}	$(T+1, T+2, \ldots, 2T, m), \ a = d = 1$
m_{T+2}	$(2T, \ldots, T+2, T+1, w)$	w_{T+2}	$(\mu(w_{T+1}), T+1, \ldots, 2T, m), \ a = d = 2$
$\vdots$			
m_{2T}	$(2T, \ldots, T+2, T+1, w)$	w_{2T}	$(\mu(w_{2T-1}), T+1, \ldots, 2T, m), \ a = d = T$
$\vdots$		$\vdots$	
$m_{(\alpha-1)T+1}$	$(\alpha T, \ldots, (\alpha-1)T+2, (\alpha-1)T+1, w)$	$w_{(\alpha-1)T+1}$	$(\mu(w_{(\alpha-1)T+1}), \ldots, \alpha T, m), \ a = d = 1$
$\vdots$		$\vdots$	

Table: Construction of Agent Preferences Used for Worst-case Substitutes Requirement in Online Matching Mechanisms