Redistribution of VCG Payments in Assignment of Heterogeneous Objects

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Outline of the Talk

- Introduction
- State of the Art and Research Gaps
- Proposed Solution
  - Experimental Analysis
- Summary and Future Work

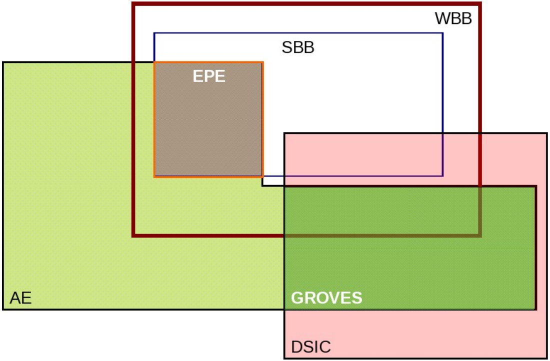
Motivation

- government body wants to allot p land properties among n of its different subdivisions
- an university wants to allot spaces to departments
- assignment of p resources among n of its users
- assignment should be such that social welfare is maximized
- we need true valuations of the agents for these objects
- *mechanism design* comes into picture

Acronyms

DSIC	Dominant Strategy Incentive Compatible
AE	Allocative Efficiency (Allocatively Efficient)
BB	Budget Balance
IR	Individual Rationality
VCG	Vickrey-Clarke-Groves Mechanisms

Green-Laffont Impossibility Theorem



AE : Allocative Efficient
DSIC : Dominant strategy Incentive Compatible
WBB : Weak Budget Balanced
SBB: Strict Budget Balanced
EPE: Ex-post efficient

Green-Laffont Impossibility Theorem [1]: AE + SBB + DSIC is not possible.

Redistribution Mechanism

- Laffont and Maskin [2] : redistribute the surplus among participating agents
- redistribute the surplus among the participating agents preserving allocative efficiency and DSIC, (Groves mechanism)
- refer to it as redistribution mechanism
- design an appropriate *rebate function*

State of the Art and Research Gaps

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$$L(n,p) = \max_{\theta \in \Theta} \frac{\text{Budget Surplus}}{\text{Efficient Surplus}}$$

- Guo and Conitzer [7] : designed mechanism which is optimal in expected sense

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Notation

θ_{ij}	The valuation of the agent i for object j
θ_i	$= (\theta_{i1}, \theta_{i2}, \ldots, \theta_{ip})$. The vector of valuations of the agent i
Θ_i	$= \mathbb{R}_+^p$. Space of valuation of agent i
Θ	$= \prod_{i \in N} \Theta_i$
b_i	$= (b_{i1}, b_{i2}, \ldots, b_{ip}) \in \Theta_i$. Bid submitted by agent i
b	$= (b_1, b_2, \ldots, b_n)$. The bid vector
K	The set of all allocations of p objects to n agents, each getting at most one object
k	An allocation, $k \in K$

$v_i(k(b))$	The valuation of the allocation to the agent i
t_i	$= v_i(k^*(b)) - \left(v(k^*(b)) - v(k^*_{-i}(b))\right)$. Payment made by agent i in VCG mechanism
t	$\sum_{i \in N} t_i$. VCG payment, total payment received from all the agents
t^{-i}	VCG payment received in absence of the agent i
r_i	Rebate to agent i
p_i	$= t_i - r_i$. Net payment made by agent i in new mechanism
Δ	$= \sum_{i \in N} p_i$. Budget imbalance in the system
e	The efficiency of the mechanism. $= \inf_{\theta : t \neq 0} \frac{\Delta}{t}$

Table: Notation

WCO Mechanism

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where,

$$f(x_1, x_2, \ldots, x_{n-1}) = \sum_{j=p+1}^{n-1} c_j x_j$$

$$c_i = \frac{(-1)^{i+p-1}(n-p)\binom{n-1}{p-1}}{i\binom{n-1}{i}\sum_{j=p}^{n-1}\binom{n-1}{j}}\left\{\sum_{j=i}^{n-1}\binom{n-1}{j}\right\};\ i = p+1, \ldots, n-1$$

Problem We Are Addressing

- p heterogeneous objects to assigned among n competing agents, where $n \geq p$ and agents have unit demand
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Goal

Design a redistribution mechanism which is individually rational, feasible and worst case optimal for assignment of p heterogeneous objects among n agents with unit demand.

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where, $L = n - p - 1$ and for $i = p + 1 \rightarrow n - 1$

$$c_i = \sum_{k=0}^{n-i-1} \alpha_{L-k} \times \frac{\binom{i-1}{p}\binom{n-i-1}{k}}{\binom{n-1}{p+1+k}} \tag{3}$$

- α's are given by,

$$\alpha_i = \frac{(-1)^{(i+1)}(L-i)!p!}{(n-i)!}\chi \sum_{j=0}^{L-i}\left\{ \binom{i+j-1}{j} \sum_{l=p+i+j}^{n-1} \binom{n-1}{l} \right\}; \quad i = 1,2,\ldots,L \tag{4}$$

where, χ is given by, $\chi = \frac{(n-p)\binom{n-1}{p-1}}{\sum_{j=p}^{n-1}\binom{n-1}{j}}$

- HETERO agrees with WCO mechanism when objects are homogeneous
- advantage : applicable even when objects are heterogeneous

Why it Works?

Conjecture

The proposed scheme, HETERO, is individually rational.

Guo and Conitzer [5] :

Theorem 1

For any $x_1 \geq x_2 \geq \ldots x_n \geq 0$,

$$a_1 x_1 + a_2 x_2 + \ldots a_n x_n \geq 0 \text{ iff } \sum_{i=1}^{j} a_i \geq 0 \;\; \forall j = 1, 2 \ldots, n.$$

1. define, $\Gamma_1 = t^{-i}$, $\Gamma_j = t^{-i,j-1}$, $j = 2, \ldots, L$
2. rebate function for agent i,

$$r = \sum_j \alpha_j \Gamma_j$$

3. note, $\Gamma_1 \geq \Gamma_2 \geq \ldots \geq \Gamma_L \geq 0$
4. for $p = 2$, $n = 4, 5, 6$; $p = 3$, $n = 5, 6, 7$; individual rationality follows from Theorem 1
5. if $\sum_{i=1}^{j} \alpha_i \geq 0 \; \forall \, j = 1 \rightarrow L$, individual rationality would follow from Theorem 1
6. Γ_j's are related
7. α_j's give appropriate weights to the combinations when a particular agent is absent in the system along with $j-1$ agents

Experiments and Empirical Evidence

Setup 1

- $p = 2, n = 5, 6, \ldots, 14,$ # Experiments $200,000$
- $p = 3, n = 7, 8, \ldots, 14,$ # Experiments $40,000$
- $p = 4, n = 9, 10, \ldots, 14,$ # Experiments $40,000$

HETERO is individually rational, feasible and performs at least as good as a worst case optimal mechanism

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Setup 2

Assume all the agents have binary valuations on each of these objects

- $p = 2, n = 5, 6, \ldots, 12$

Enumerate all possible bids.

HETERO is individually rational, feasible, and worst case optimal

Summary

- a redistribution mechanism (HETERO): assigns p heterogeneous objects among n agents with unit demand
- it is individually rational for particular combinations of n and p
- experimental analysis : it is individually rational, feasible and performs equally good as a worst case optimal mechanism
- the agents with binary valuations : for some combinations of n and p, it is individually rational, feasible and worst case optimal

Directions for Future Work

Ongoing work

- prove HETERO is individually rational when $p = 2$
- feasibility
- worst case analysis and design of worst case optimal mechanism
- all of the above when $p > 2$

Future work

- extensions to multi-unit demands
- linear rebate function : linear in received bids
- no redistribution mechanism with linear rebate function that redistributes non-zero fraction of VCG surplus in the worst case
    - characterize situations under which linear rebate functions that redistribute non-zero fraction of the VCG surplus even in the worst case

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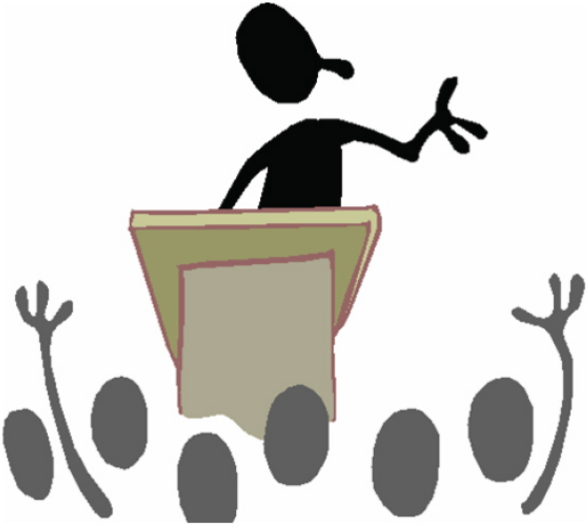
Questions?



Thank You!!!